Practice problems

Problem 1
Diseases I and II are common among people in a certain population. It is assumed that 10% of the population will contract disease I sometime during their lifetime, 15% will contract diesease II, and 3% will contract both diseases.

a. Find the probability that a randomly chosen person from this population will contract at least one disease?

b. Find the conditional probability that a randomly chosen person from this population will contract both diseases, given that he or she has contracted at least one disease.

c. Are the events "contracting disease I" and "contracting disease II" independent?

Problem 2
Part A:
Answer the following questions:

a. You toss simultaneously 3 fair coins until all three show the same face. What is the probability that all three coins show the same face on the third attempt?

b. Let events A, B. What probability does the expression $P(A) + P(B) - 2P(A \cap B)$ represent?

c. You randomly select 2 cards without replacement from a standard 52-card deck that has 13 clubs ($\clubsuit$), 13 spades ($\spadesuit$), 13 diamonds ($\diamondsuit$), and 13 hearts ($\heartsuit$). What is the probability that both cards are of the same suit?

Part B:
Let events A, B. Show that

$$P(A|B) + P(A'|B) = 1$$

Problem 3
Part A:
Indicate (without computation) which list has the higher standard deviation:

a. List A: 20, 20, 20, 20, 20
List B: 20, 20, 20, 20, 19

b. List A: 20, 25, 25, 25, 30
List B: 15, 25, 25, 25, 35

c. List A: 20, 20, 30, 40, 40
List B: 20, 25, 30, 35, 40

d. List A: 1,1,1,2,2,2,3,3,3,4,4,4
List B: 1,1,1,1,2,2,2,2,3,3,3,3,4,4,4,4

Part B:
For a sample of size $n = 50$ you are given the following: $\sum_{i=1}^{50} x_i^2 = 100, \bar{x} = 0.8$. If you have enough information please compute the sample variance.

Problem 4
Answer the following questions:

a. Five cards are selected without replacement from an ordinary 52-card deck. Find the probability that you obtain at least 1 clubs.

b. For two events A, B it is given that $P(A) = 0.3, P(B) = 0.6, P(A \cap B) = 0.1$. Find the probability that none of these two events occurs.

c. Two dice are thrown n times in succession. How large need n be so that the probability of at least one double six is at least $\frac{1}{2}$.

d. Two dice are rolled and the sum of the two numbers is observed. Given that the sum is 10 what is the probability that the double five occurred?

e. In the game of graps two dice are rolled and the sum is observed. If a player rolls a sum of 5 then he will have to roll the two dice again until the sum of 5 or the sum of 7 is observed. If the sum of 5 is observed before the sum of 7 the player wins. If the sum of 7 is observed before the sum of 5 the player loses. A player rolls the sum of 5 on the first trial. What is the probability that he wins on the 5th trial?

Problem 5
Part A:
Bowl B_1 contains 3 white and 9 green chips. Bowl B_2 contains 8 white and 4 green chips. Bowl B_3 contains 10 white and 2 green chips. A bowl is selected at random (with equal probabilities), and one chip is selected.

a. Compute the probability of selecting a white chip.

b. Suppose a bowl was selected and handed to you without your being told which bowl you hold. If you select a white chip from this bowl, what is the conditional probability that you were handed bowl B_3?

Part B:
Two events A and B are such that $P(A) = 0.2$, $P(B) = 0.3$ and $P(A \cup B) = 0.4$.

a. Find $P(A \cap B)$.

b. Find $P(A' \cup B')$.

c. Find $P(A'|B)$.

Problem 6
Part A:
For a gambling game if a player bets \$1 his expected profit is $E(X) = -\$0.053$ with variance $Var(X) = \$33.21$. What is his expected profit and variance of the following two situations:

a. The player will bet \$40 and play the game once.

b. The player will play the game 40 times and will bet each time \$1.

Part B:
In a gambling game a player who draws a jack or a queen is paid \$15 and \$5 for drawing a king or an ace from an ordinary deck of fifty-two playing cards. A player who draws any other card pays \$4. Let X be the player's profit. Find the expected value and variance of X.

Problem 7
In a bolt factory machines A, B, C manufacture, respectively, 25, 35, and 40 percent of the total. It is known that of their output 5, 4, and 2 per cent are defective bolts. The bolts are shipped to warehouses and suppose that a bolt is drawn at random from a certain warehouse and is found defective. What is the probability it was *not* manufactured by machine A?