

University of California, Los Angeles  
Department of Statistics

Statistics 100A

Instructor: Nicolas Christou

Exam 1  
17 July 2007

Name: \_\_\_\_\_

**Problem 1 (20 points)**

**Part A:**

Use the binomial theorem to show that:

a.  $\sum_{k=0}^n \binom{n}{k} (-1)^k = 0.$

b.  $\sum_{k=0}^n \binom{n}{k} (a - 1)^k = a^n.$

**Part B:**

What is the probability that among 5 people at least two have their birthday in the same month?

**Problem 2 (20 points)**

Consider a two-stage game. At the first stage, we flip a fair coin. If the coin comes up heads, we select one ball from urn 1, and if the coin comes up tails, we select a ball from urn 2. The urns have the following composition:

Urn 1: 5 green balls and 10 yellow balls.

Urn 2: 20 green balls and 10 yellow balls.

You arrived late for this game, and you missed the coin flip. However, you did get to witness the selection of a yellow ball. What is the probability that the ball came from urn 1?

**Problem 3 (20 points)**

A certain company produces air filters at three different assembly plants. The first plant makes 60% of all the filters, and 1% of its filters will be returned by customers because of defects. The second plant makes 30% of all the filters, and 2% of its filters will be returned by customers because of defects. The third plant makes 10% of all the filters, and 3% of its filters will be returned by customers because of defects. In the process of packing the filters for shipment to retail outlets, the filters from the three plants are intermixed.

- a. Suppose a filter is randomly selected from a particular retail outlet. What is the probability that the filter will be found defective.

- b. Suppose that a filter was returned (it was found defective). What is the probability that this filter was made at the first plant?

**Problem 4 (20 points)**

**Part A:**

A man buys a racehorse for \$20000, and enters it in two races. He plans to sell the horse afterward, hoping to make a profit. If the horse wins both races, its value will jump to \$100000. If it wins one of the races, it will worth \$50000. If it loses both races, it will worth only \$10000. The man believes there is a 20% chance that the horse will win the first race and a 30% chance it will win the second one.

a. Assuming that the two races are independent events, find the man's expected profit.

b. Find the standard deviation of the man's profit.

**Part B:**

Suppose that  $X$  takes on one of the values 0, 1, 2. If for some constant  $c$ ,  $P(X = i) = cP(X = i - 1), i = 1, 2$  find  $E(X)$  in terms of  $c$ .

**Problem 5 (20 points)**

Let  $X$  be a discrete random variable which has the following probability distribution.

$X$	$P(X)$
0	0.2
1	0.3
2	0.4
3	0.1

a. Compute  $E(4 + X)$ .

b. Compute  $\text{Var}(X)$ .

c. Compute  $E(X^2 - 2X + 1)$ .

d. Let  $Y = 0.4X - 2$ . Compute  $\text{Var}(Y)$ .