**University of California, Los Angeles**
**Department of Statistics**

**Statistics 100A** **Instructor: Nicolas Christou**

**Exam 1**
**23 October 2007**

**Name:** ______________________________

**Problem 1 (25 points)**
Answer the following questions:

a. What is the probability that you and the person sitting next to you have the same birthday?
*Answer:*
$$1 - \frac{365 \times 364}{365^2} = \frac{1}{365}.$$

b. What is the probability that the 52 cards will be divided into 3 piles of 15, 18, and 19 cards each, such that the first pile gets 3 black cards, the second pile gets 10 black cards, and the third pile gets 13 black cards?
*Answer:*
$$\binom{26}{3, 10, 13} \times \binom{26}{12, 8, 6}.$$

c. Five cards will be selected without replacement from the 52-card deck. What is the probability that all five cards are spades?
*Answer:*
$$\frac{\binom{13}{5}\binom{39}{0}}{\binom{52}{5}} = \frac{\binom{13}{5}}{\binom{52}{5}}.$$

d. Five cards will be selected without replacement from the 52-card deck. What is the probability that we get 2 aces, 2 tens, and one five?
*Answer:*
$$\frac{\binom{4}{2}\binom{4}{2}\binom{4}{1}}{\binom{52}{5}}.$$

e. A box contains 2 apples, 2 oranges, 2 nectarines, 2 peaches, and 2 pears. Three fruits are selected at random without replacement. Find the probability that among the three at least 1 is an apple.
*Answer:*
$$\frac{\binom{2}{1}\binom{8}{2} + \binom{2}{2}\binom{8}{1}}{\binom{10}{3}}.$$

**Problem 2 (25 points)**
Bowl $B_1$ contains 3 white and 9 green chips. Bowl $B_2$ contains 8 white and 4 green chips. Bowl $B_3$ contains 10 white and 2 green chips. A card is drawn from an ordinary 52-card deck. If a face card is drawn (king, queen, jack), a chip is selected from bowl $B_1$. If an ace is drawn, a chip is selected from bowl $B_2$. If any other card is drawn, a chip is selected from bowl $B_3$.

a. What are the prior (unconditional) probabilities of selecting each bowl?
*Answer:*
$P(B_1) = \frac{12}{52}, P(B_2) = \frac{4}{52}, P(B_3) = \frac{36}{52}$.

b. Compute the probability of selecting a white chip.
*Answer:*

$$\begin{aligned} P(W) &= P(W \cap B_1) + P(W \cap B_2) + P(W \cap B_3) \\ &= P(W|B_1)P(B_1) + P(W|B_2)P(B_2) + P(W|B_3)P(B_3) \\ &= \frac{3}{12} \times \frac{12}{52} + \frac{8}{12} \times \frac{4}{52} + \frac{10}{12} \times \frac{36}{52}. \end{aligned}$$

c. Suppose a bowl was selected and handed to you without you being told which bowl you hold. If you select a white chip from this bowl, what is the posterior (conditional) probability that you were handed bowl $B_1$, $B_2$, $B_3$?
*Answer:*

$$\begin{aligned} P(B_1|W) &= \frac{P(B_1 \cap W)}{P(W)} = \ldots \\ P(B_2|W) &= \frac{P(B_2 \cap W)}{P(W)} = \ldots \\ P(B_2|W) &= \frac{P(B_3 \cap W)}{P(W)} = \ldots \end{aligned}$$

Note: $P(W)$ was found in part (b). The numerator for each of the expressions above was also calculated in part (b).

**Problem 3 (25 points)**
A discrete random variable has the following probability:

| $X$ | $P(X)$ |
|---|---|
| -3 | $\frac{1}{4}$ |
| -1 | $\frac{1}{8}$ |
| 0 | $\frac{3}{8}$ |
| 2 | $\frac{1}{8}$ |
| 3 | $\frac{1}{8}$ |

Answer the following questions:

a. Find $E(X)$.
*Answer:*

$$E(X) = \mu = \sum_x xp(x) = -3(\frac{1}{4}) - 1(\frac{1}{8}) + 0(\frac{3}{8}) + 2(\frac{1}{8}) + 3(\frac{1}{8}) = -\frac{1}{4}.$$

b. Find $Var(X)$.
*Answer:*

$$\begin{aligned} Var(X) &= \sigma^2 = \sum_x x^2 p(x) - \mu^2 \\ &= 9(\frac{1}{4}) + 1(\frac{1}{8}) + 0(\frac{3}{8}) + 4(\frac{1}{8}) + 9(\frac{1}{8}) - \frac{1}{16} = \frac{63}{16}. \end{aligned}$$

c. Use (a) and (b) to compute $E(2X - 3X^2)$.
*Answer:*

$$E(2X - 3X^2) = 2E(X) - 3EX^2 = 2E(X) - 3(\sigma^2 + \mu^2) = \ldots$$

d. Find $P(X = 0|X < 2)$.
*Answer:*

$$P(X = 0|X < 2) = \frac{P(X = 0 \cap X < 2)}{P(X < 0)} = \frac{P(X = 0)}{P(X < 2)} = \frac{\frac{3}{8}}{\frac{1}{8} + \frac{3}{8} + \frac{1}{8} + \frac{1}{4}}.$$

**Problem 4 (25 points)**

**Part A:**

A player will play the roulette game and will bet \$40 on a single number that pays 35 to 1. What is his expected profit and variance.

*Answer:*

| $X$ | $P(X)$ |
|---|---|
| 35 | $\frac{1}{38}$ |
| -1 | $\frac{37}{38}$ |

$E(X) = 35(\frac{1}{38}) - 1(\frac{37}{38}) = -\$0.053$. When the player bets \$40 on a single game the expected profit will be: $E(40X) = 40(-0.053) = -2.12$.

$Var(X) = 35^2(\frac{1}{38}) + 1^2(\frac{37}{38}) - 0.053^2 = 33.56$. When the player bets \$40 on a single game the variance will be: $Var(40X) = 40^2(33.56) = 53696$.

**Part B:**

A player will play the roulette game 40 times and will bet each time \$1 on a single number that pays 35 to 1. What is his expected profit and variance.

*Answer:*

$E(X_1 + \ldots + X_{40}) = 40(-0.053) = -2.12$.

$Va(X_1 + \ldots + X_{40}) = 40(33.56) = 1342.4$.

**Part C:**

The events $A, B$ of an experiment are mutually exclusive with probabilities $P(A) = 0.5, P(B) = 0.3$. The experiment is performed independently until one of the two events will occur. What is the probability that event $A$ will occur before event $B$.

*Answer:*

$$P(A|A \text{ or } B) = \frac{P(A)}{P(A \text{ or } B)} = \frac{0.5}{0.5 + 0.3} = \frac{5}{8}.$$