University of California, Los Angeles
Department of Statistics

Statistics 100A **Instructor: Nicolas Christou**

Exam 2
16 November 2010

Name: _____

Problem 1 (25 points)
Answer the following questions:

a. Many people cancel their reservations at hotels at the last minute. Therefore, most hotels overbook when possible. That is, they make more reservations that they have rooms. A particular hotel has 90 rooms, and, on the average, 20% of the people with reservations cancel them. Suppose that for a particular night the hotel has 100 reservations. Approximate the probability that more people will show up with reservations than they have rooms for?

b. Suppose that accidents on a particular road occur as a Poisson process at a rate of two per month. Approximate the probability that in 18 months there will be more than 30 accidents?

Problem 2 (25 points)
Answer the following questions:

a. The graph on the previous page shows three gamma densities, $\Gamma(3,1), \Gamma(10,1)$, and $\Gamma(1,1)$. Please identify which one is which.

b. Suppose that X follows the uniform distribution on (0, n). Find the mean and variance of X.

c. Refer to question (b). Show that

$$Y = \frac{X - \mu_X}{\sigma_X}$$

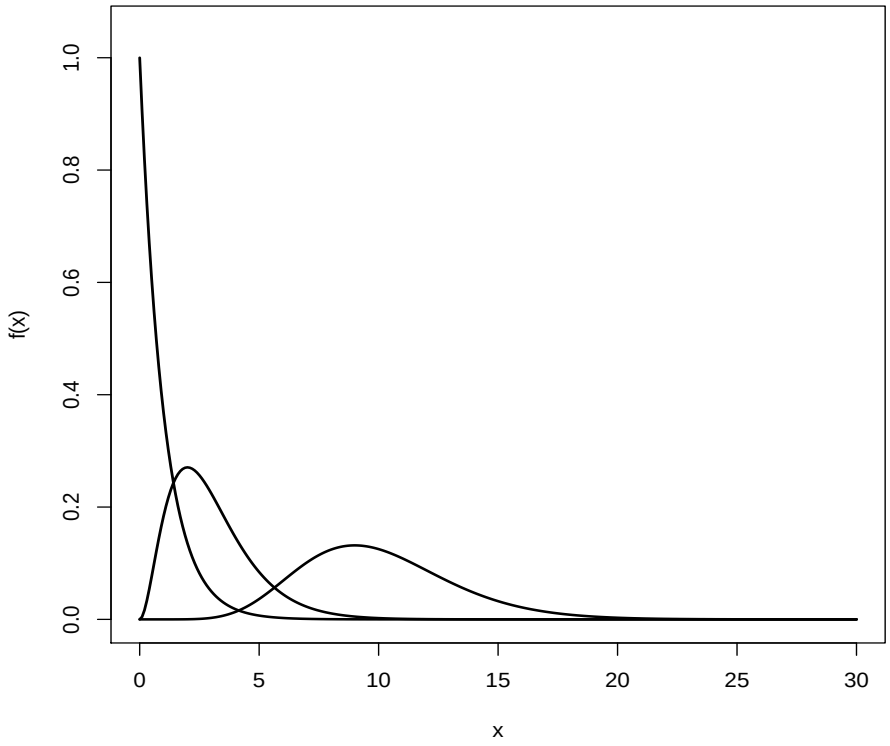
follows also the uniform distribution on the interval $(-\frac{\sqrt{12}}{2}, \frac{\sqrt{12}}{2})$ with pdf $f(y) = \frac{1}{\sqrt{12}}$.

d. Since the standard normal density function integrates to 1 we should have:

$$2\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz = 1.$$

By using $u = \frac{1}{2}z^2$ show that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

Problem 2 - Question (a):



Problem 3 (25 points)
Answer the following questions:

a. Let $X \sim \Gamma(\alpha, \beta)$. Find EX^{-1}. (The gamma distribution is one of the few distributions for which EX^{-1} can be computed easily).

b. Let $X \sim \Gamma(\alpha, \beta)$. Show that $Y = \frac{X}{\beta}$ follows $\Gamma(\alpha, 1)$.

c. Suppose people move into a town as a Poisson process with parameter $\lambda = 5$ per week. Use gamma to write the expression that computes the probability that the 64th person will move to this town in less than 15 weeks.

d. Approximate the probability of part (c) using the normal distribution.

e. Use Poisson to write the expression that computes the exact probability of question (c).

Problem 4 (25 points)
The pdf of a continuous random variable X is $f(x) = 2x$ for $0 \leq x \leq 1$.

a. Find the cdf of X.

b. Find the 81st percentile of X.

c. Find the mean and variance of X.

d. Let $y_1, y_2, \ldots, y_n$ be the values of a random sample taken from the uniform (0,1) distribution. Use the inverse transformation method to explain how you can generate random values from $f(x) = 2x$.