University of California, Los Angeles
Department of Statistics

Statistics 100A **Instructor: Nicolas Christou**

Exam 2
13 May 2011

Name: ____________________________

Problem 1 (25 points)
Answer the following questions:

a. Let $X \sim \Gamma(\alpha, \beta)$. Show that $Y = cX$ follows $\Gamma(\alpha, c\beta)$.

b. Let $X \sim N(\mu, \sigma)$. Find the distribution of $Y = e^X$.

c. Suppose the radius of a circle X is a random variable that follows the exponential distribution with parameter λ. Find the distribution of the area of the circle: $Y = \pi X^2$.

Problem 2 (25 points)
In California earthquakes of magnitude 1-2 in the Richter scale are recorded at the rate of 8 per hour according to a Poisson distribution. Answer the following questions:

a. What is the probability that more than 12 earthquakes (of magnitude 1-2 in the Richter scale) will be recorded in the next hour. Please write the expression that computes the exact probability (no computations).

b. Approximate the probability of part (a) using the normal distribution.

c. What is the distribution of the time that you have to wait until the 30th earthquake (of magnitude 1-2 in the Richter scale) is recorded. Please write the complete density of the distribution.

d. Using the distribution of part (c) write the expression that computes the probability that the 30th earthquake (of magnitude 1-2 in the Richter scale) will be recorded in less than 4 hours from now.

e. Approximate the probability of part (d).

f. Write the expression that computes the same probability of part (d) using the Poisson distribution (no computations).

Problem 3 (25 points)
Answer the following questions:

a. Using your class notes and the one-page handout on the beta distribution show that the variance of beta distribution is

$$var(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}.$$

b. Scores on a certain standardized test, IQ scores, follow the normal distribution with mean $\mu = 100$ and standard deviation $\sigma = 13$. An individual is selected at random. What is the probability that his score satisfies $120 < X < 130$?

c. Refer to part (b). Find the 40th percentile of the distribution of these scores.

Problem 4 (25 points)
Answer the following questions:

a. It is said that a random variable X follows the Pareto distribution with parameters x_0 and α with $x_0 > 0, \alpha > 0$, if X has the following probability density function:

$$f(x) = \frac{\alpha x_0^{\alpha}}{x^{\alpha+1}}, \quad \text{for } x \geq x_0.$$

Show that $Y = ln(\frac{x}{x_0})$ follows the exponential distribution with parameter α. Note: x_0 is a constant.

b. Let $X \sim \Gamma(\alpha, \beta)$, with $\alpha > 2, \beta > 0$. Show that the variance of $\frac{1}{X}$ is $\frac{1}{\beta^2(\alpha-1)^2(\alpha-2)}$. *Hint*: See your class notes for EX^k.

c. Let U be a uniform random variable on [0,1], and let $V = \frac{1}{U}$. Find the probability density function of V. For what values of V is this density valid? On the previous page please draw the density of U and the density of V on two separate graphs.