

University of California, Los Angeles
Department of Statistics

Statistics 100A

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Homework 1

(Some are from: Sheldon Ross (2002), *A first Course in Probability*, Sixth Edition, Prentice Hall).

EXERCISE 1

Suppose that n people are seated in a random manner in a row of n theater seats. In how many ways can two particular people A and B will be seated next to each other?

EXERCISE 2

If k people are seated in a random manner in a row containing n seats ($n > k$), in how many ways can the people will occupy k adjacent seats?

EXERCISE 3

Suppose that a committee of 12 people is selected in a random manner from a group of 100 people. Determine the number of ways that two particular people A and B will both be selected.

EXERCISE 4

Prove that

$$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$$

EXERCISE 5

Prove that

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n.$$

Hint: Use the binomial theorem.

EXERCISE 6

Prove that

$$\binom{n+m}{r} = \binom{n}{0} \binom{m}{r} + \binom{n}{1} \binom{m}{r-1} + \cdots + \binom{n}{r} \binom{m}{0}.$$

EXERCISE 7

Use the result of exercise 6 to prove that

$$\binom{2n}{n} = \sum_{k=0}^n \binom{n}{k}^2.$$

EXERCISE 8

From a group of n people, suppose that we want to choose a committee of k , $k \leq n$, one of whom is to be designated as chairperson.

- a. By focusing first on the choice of the committee and then on the choice of the chair, argue that there are $\binom{n}{k}k$ possible choices.
- b. By focusing first on the choice of the nonchair committee members and then on the choice of the chair, argue that there are $\binom{n}{k-1}(n-k+1)$ possible choices.
- c. By focusing first on the choice of the chair and then on the choice of the other committee members, argue that there are $n\binom{n-1}{k-1}$ possible choices.

EXERCISE 9

Gambling (only for here...)

As a reminder a standard 52-card deck has $13\clubsuit$, $13\spadesuit$, $13\diamondsuit$, and $13\heartsuit$. Also in these 52 cards there are 4 cards of each of $A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K$, where A is an ace, J is a jack, Q is queen, and K is a king. A poker hand consists of 5 cards. If the cards have distinct consecutive values and are not all of the same suit, we say that the hand is a straight. For example, the following is a straight: $6\clubsuit, 7\clubsuit, 8\clubsuit, 9\clubsuit, 10\spadesuit$. The ace plays either high or low which means that $A\clubsuit, 2\spadesuit, 3\heartsuit, 4\diamondsuit, 5\clubsuit$ and $10\spadesuit, J\heartsuit, Q\clubsuit, K\diamondsuit, A\diamondsuit$ are valid straights. However “around the corner” straights like $J\heartsuit, Q\clubsuit, K\clubsuit, A\clubsuit, 2\spadesuit$ are not allowed. In how many ways can a player receive a straight?

EXERCISE 10

More gambling (again only for here...)

A 5-card poker hand is said to be a full house if it consists of 3 cards of the same number and 2 cards of the same number (3 of kind plus 2 of a kind). For example, the following is a full house: $2\clubsuit, 2\spadesuit, 2\heartsuit, 8\clubsuit, 8\diamondsuit$. In how many ways can a player receive a full house?

EXERCISE 11

Suppose that a deck of 25 cards contains 12 green cards. Suppose also that the 25 cards are distributed in a random manner to three players A, B , and C in such a way that player A receives 10 cards, player B receives 8 cards, and player C receives 7 cards. Determine the number of ways that player A will receive 6 green cards, player B will receive 2 green cards, and player C will receive 4 green cards.