University of California, Los Angeles
Department of Statistics

Statistics 100A **Instructor: Nicolas Christou**

Homework 1

(Some of these problems are from: Sheldon Ross (2002), *A first Course in Probability*, Sixth Edition, Prentice Hall).

EXERCISE 1
Suppose that n people are seated in a random manner in a row of n theater seats. In how many ways can two particular people A and B will be seated next to each other?

EXERCISE 2
If k people are seated in a random manner in a row containing n seats $(n > k)$, in how many ways can the people will occupy k adjacent seats?

EXERCISE 3
Suppose that a committee of 12 people is selected in a random manner from a group of 100 people. Determine the number of ways that two particular people A and B will both be selected.

EXERCISE 4
Prove that

$$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$$

EXERCISE 5
Prove that

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n.$$

Hint: Use the binomial theorem.

EXERCISE 6
Prove that

$$\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \cdots \binom{n}{r}\binom{m}{0}.$$

EXERCISE 7
Use the result of exercise 6 to prove that

$$\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2.$$

EXERCISE 8
From a group of n people, suppose that we want to choose a committee of k, $k \leq n$, one of whom is to be designated as chairperson.

a. By focusing first on the choice of the committee and then on the choice of the chair, argue that there are $\binom{n}{k}k$ possible choices.

b. By focusing first on the choice of the nonchair committee members and then on the choice of the chair, argue that there are $\binom{n}{k-1}(n-k+1)$ possible choices.

c. By focusing first on the choice of the chair and then on the choice of the other committee members, argue that there are $n\binom{n-1}{k-1}$ possible choices.

EXERCISE 9

Gambling (only for here...)

As a reminder a standard 52-card deck has 13$\clubsuit$, 13$\spadesuit$, 13$\diamondsuit$, and 13$\heartsuit$. Also in these 52 cards there are 4 cards of each of $A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K$, where A is an ace, J is a jack, Q is queen, and K is a king. A poker hand consists of 5 cards. If the cards have distinct consecutive values and are not all of the same suit, we say that the hand is a straight. For example, the following is a straight: $6\clubsuit, 7\clubsuit, 8\clubsuit, 9\clubsuit, 10\spadesuit$. The ace plays either high or low which means that $A\clubsuit, 2\spadesuit, 3\heartsuit, 4\diamondsuit, 5\clubsuit$ and $10\spadesuit, J\heartsuit, Q\clubsuit, K\diamondsuit, A\diamondsuit$ are valid straights. However "around the corner" straights like $J\heartsuit, Q\clubsuit, K\clubsuit, A\clubsuit, 2\spadesuit$ are not allowed. In how many ways can a player receive a straight?

EXERCISE 10

More gambling (again only for here...)

A 5-card poker hand is said to be a full house if it consists of 3 cards of the same number and 2 cards of the same number (3 of kind plus 2 of a kind). For example, the following is a full house: $2\clubsuit, 2\spadesuit, 2\heartsuit, 8\clubsuit, 8\diamondsuit$. In how many ways can a player receive a full house?

EXERCISE 11

Suppose that a deck of 25 cards contains 12 green cards. Suppose also that the 25 cards are distributed in a random manner to three players A, B, and C in such a way that player A receives 10 cards, player B receives 8 cards, and player C receives 7 cards. Determine the number of ways that player A will receive 6 green cards, player B will receive 2 green cards, and player C will receive 4 green cars.