University of California, Los Angeles
Department of Statistics

Statistics 100A **Instructor: Nicolas Christou**

Homework 2

EXERCISE 1

The Statistics Society at a large university would like to determine whether there is a relationship between a student's interest in statistics and his or her ability in mathematics. A random sample of 200 students is selected and they are asked whether their ability in mathematics and interest in statistics is low, average, or high. The results were as follows.

		Ability in Mathematics			
		Low	Average	High	Total
Interest in Statistics	Low	60	15	15	90
	Average	15	45	10	70
	High	5	10	25	40
	Total	80	70	50	200

A person is randomly selected.

a. What is the probability that the person has high interest in statistics?

b. What is the probability that the person has high ability in mathematics and high interest in statistics?

c. Given that the person has high ability in mathematics, what is the probability that the person has high interest in statistics?

d. Are the events "high ability in mathematics" and "high interest in statistics" independent?

EXERCISE 2

Suppose a woman tries on a dress. The probability that she asks for alterations is 0.65. The probability that she asks for delivery is 0.32. The probability that she asks for alterations and delivery is 0.21. Suppose that you randomly select a woman who has tried on a dress. What is the probability that:

a. She will either ask for alterations or for delivery of the dress or both.

b. She will not ask for alterations and she will not ask for delivery of the dress.

EXERCISE 3

A tennis player A has probability of $\frac{2}{3}$ of winning a set against player B. A match is won by the player who first wins three sets. Find the probability that A wins the match.

EXERCISE 4

In a game of chance each player throws two unbiased dice, each one numbered 1,2,3,4,5,6, and scores the difference between the larger and smaller numbers which arise. Two players compete and one or the other wins if, and only if, he scores at least 4 more than his opponent.
Find the probability that neither player wins.

EXERCISE 5

Observations of a waiting line at a medical clinic indicates that the probability that a new arrival will be an emergency case is $p = \frac{1}{6}$. Find the probability that the r_{th} patient is the first emergency case. Assume that conditions of arriving patients represent independent events.

EXERCISE 6

Three identical fair coins are thrown simultaneously until all three show the same face. What is the probability that they are thrown more than three times.

EXERCISE 7
There are 3 coins in a box. One is a two-headed coin; another is a fair coin; and the third is a biased coin that comes heads 75% of the time.

a. When one of the 3 coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin?

b. The same coin is flipped again and it shows heads. What is the probability that it is the fair coin?

c. The coin is placed back in the box. When one of the 3 coins is selected at random and flipped twice, it shows tails in both tosses. What is the probability that it was the biased coin?

EXERCISE 8
Given that a person has a certain disease, a diagnostic test will detect it with probability 0.90. Also, given that a person does not have the disease, the diagnostic test will detect that the person has the disease with probability 0.10. Only 1% of the population has the disease in question.
You can use:
T= {diagnostic test detects that a person has the disease}
D= {person actually has the disease}

a. A person is randomly selected from the population and tested for this disease. What is the probability that the diagnostic test will detect that the person has the disease?

b. A person is chosen at random from the population. Given that the diagnostic test detects that the person has the disease, what is the probability that the person actually has the disease?

EXERCISE 9
Three people A, B, and C gamble for a prize by rolling a die. The first to roll a five wins. If the die is unbiased (fair) and they roll in order A, then B, then C, find the probability that :

a. A wins on his first throw.

b. C wins at his first attempt.

c. B wins at his third attempt.

d. A wins.

EXERCISE 10
Five identical bowls are labeled 1,2,3,4, and 5. Bowl i contains i white balls and $5 - i$ black balls, $i = 1, 2, 3, 4, 5,$. A bowl is randomly selected and two balls are selected without replacement from the contents of the bowl.

a. What is the probability that both balls selected are white?

b. Given that both balls selected are white, what is the probability that bowl 3 was selected?

EXERCISE 11
Use the distributive law, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, to prove that:
$P[A \cup B)|C] = P(A|C) + P(B|C) - P(A \cap B|C)$. Suppose that A, B, and C are mutually exclusive events. What will change in the right-hand side of the previous equation?

EXERCISE 12
Let A, B, and C be three arbitrary events. Show that the probability that exactly one of these three events will occur is

$$P(A) + P(B) + P(C) - 2P(A \cap B) - 2P(A \cap C) - 2P(B \cap C) + 3P(A \cap B \cap C)$$

EXERCISE 13
An urn contains 25 balls numbered from 1 to 25. Four balls are randomly selected without replacement. Let X_1, X_2, X_3, X_4 be the numbers of the balls drawn,

a. Find the probability $\max(X_1, X_2, X_3, X_4) < 10$.

b. Find the probability $\min(X_1, X_2, X_3, X_4) < 10$.