University of California, Los Angeles
Department of Statistics

Statistics 100A **Instructor: Nicolas Christou**

Homework 3

EXERCISE 1
Use the binomial theorem (go back to your classnotes from the beginning of the course) to show that if $X \sim b(n,p)$ then $\sum_{x=0}^{n} p(x) = 1$.

EXERCISE 2
New York Lotto is played as follows: Out of 59 numbers 6 are chosen at random without replacement. Then from the remaining 53 numbers 1 is chosen. This last number is called "the bonus number". You, the player, select 6 numbers. To win the first prize you must match your 6 numbers with the State's 6 numbers. If you match only 5 numbers and your 6_{th} number matches the bonus number then you win the second prize.

a. What is the probability of winning the first prize?

b. What is the probability of winning the second prize?

c. What is the probability of winning a prize (either the first or the second)?

Note: Check your answers to (a,b) at *http://www.nylottery.org.*

EXERCISE 3
A satellite system consists of n components and functions on any given day if at least k of the n components function on that day. On a rainy day each of the components independently functions with probability p_1, whereas on a dry day they independently function with probability p_2. The probability of a rainy day is θ.

a. Give an expression of the probability that the satellite system will function at any given day.

b. Evaluate the above probability when $n = 3$, $k = 1$, $p_1 = 0.85$, $p_2 = 0.90$, and $\theta = 0.20$.

c. If the satellite system functioned yesterday what is the probabilty that it was a rainy day? Use $n = 3$, $k = 1$, $p_1 = 0.85$, $p_2 = 0.90$, and $\theta = 0.20$.

EXERCISE 4
Let X be a geometric random variable with probability of success p.

a. Show that for a positive integer k, $P(X > k) = (1-p)^k$.

b. Show that

$$P(X > n+k-1 | X > n-1) = P(X > k)$$

EXERCISE 5
Two boys play in the following way. They take turns shooting and stop when a basket is made. Player A goes first and has probability p_1 of making a basket on any throw. Player B, who shoots second, has probability p_2 of making a basket. The outcomes of succesive outcomes are assumed to be independent.

a. Find the probability mass function for the total number of attempts.

b. What is the probability that player A wins?

EXERCISE 6
The probability of being dealt a royal flush (ace, king, queen, jack, and ten of the same suit) in poker is 0.00000154. Suppose that a poker player sees 100 hands per week, 52 weeks a year, for 20 years.

a. What is the probability that this player never sees a royal flush dealt?

b. What is the probability that this player sees 2 royal flushes dealt?

EXERCISE 7
Suppose that a discrete random variable X has the following probability mass function:

$$f(x) = \begin{cases} \frac{c}{x^2} & 1, 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases}$$

Find the constant c.

EXERCISE 8
Part a:
Let X be a Poisson random variable with parameter λ. Show that $E(X) = \lambda$. *Hint:* Use the same trick we used in class for the binomial distribution.
Part b:
Suppose that in a city the number of suicides can be approximated by a Poisson process with $\lambda = 0.33$ per month.

a. Find the probability of k suicides in a year for $k = 0, 1, 2, \cdots$. What is the most probable number?

b. What is the probability of two suicides in one week?

EXERCISE 9
Suppose that three identical fair coins are thrown simultaneously. What is the probability that the 10_{th} trial is the 9_{th} time that all three show the same face?

EXERCISE 10
Let $X \sim b(n, p)$. Find the third moment, that is $E(X^3)$. *Hint:* Start with $E[X(X-1)(X-2)]$ and use the same trick we used in class for the binomial distribution.

EXERCISE 11
Suppose a fair die is rolled 20 times. What is the probability that 1 appears five times, 2 and 3 four times each, 4 and 5 three times each, and 6 once?

EXERCISE 12
An urn contains 200 marbles of which 150 are green and 50 are blue. Five marbles are selected without replacement from this urn. Let X be the number of green marbles among the five selected.

a. Use `SOCR` to construct the exact distribution of X. Submit a printout.

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