University of California, Los Angeles
Department of Statistics

Statistics 100A **Instructor: Nicolas Christou**

Homework 4

EXERCISE 1
The life of a certain type of automobile tire is normally distributed with mean 34000 miles and standard deviation 4000 miles.

a. What is the probability that such a tire lasts over 40000 miles?

b. Given that one of these tires has survived more than 30000 miles, what is the conditional probability that it survives for more than another 10000 miles?

c. Find the probability that 2 out of 5 randomly selected tires will have lifetime 1.96 standard deviations above the mean.

EXERCISE 2
Let X be a random variable with probability density function

$$f(x) = \begin{cases} c(1-x^2)^2 & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

a. What is the value of c?

b. Find the cumulative distribution function of X?

EXERCISE 3
A filling station is supplied with gasoline once a week. Its weekly volume of sales in thousands of gallons is a random variable with probability density function $f(x) = c(1-x)^4$, for $0 < x < 1$, and $f(x) = 0$ otherwise.

a. What is the value of c?

b. What need the capacity of the tank be so that the probability of the supply's being exhausted in a given week is 1%?

EXERCISE 4
The length of the tails of a certain race of dogs follows the normal distribution with mean μ and standard deviation σ . It is known that 5% of the tails is longer than 12 inches. It is also known that 2.5% of the tails is shorter than 7 inches.

a. Find μ and σ.

b. Suppose that a dog is randomly selected. What is the probability that the length of its tail will be longer than 11 inches?

EXERCISE 5
Let $F(x) = 1 - exp(-\alpha x^\beta)$ for $x \geq 0, \alpha > 0, \beta > 0$, and $F(x) = 0$ for $x < 0$. Show that F is a cumulative distribution function, and find the corresponding density.

EXERCISE 6
Let $f(x) = \frac{1+\alpha x}{2}$ for $-1 \leq x \leq 1$ and $f(x) = 0$ otherwise, where $-1 \leq \alpha \leq 1$. Show that f is a probability density function, and find the corresponding cumulative distribution function. Find the quartiles and the median of the distribution in terms of α.

EXERCISE 7
Suppose that X has the probability density function $f(x) = cx^2$ for $0 \leq x \leq 1$ and $f(x) = 0$ otherwise.

a. Find the constant c.

b. Find the cumulative distribution function.

c. What is $P(0.1 \leq X \leq 0.5)$?

EXERCISE 8
Find the lower and upper quartiles (25_{th} and 75_{th} percentiles) of the exponential distribution.

EXERCISE 9
Suppose that the lifetime of an electronic component follows an exponential disribution with parameter $\lambda = 0.1$.

a. Find the probability that the lifetime is less than 10.

b. Find the probability that the lifetime is between 5 and 15.

c. Find t such that the lifetime is greater that t is 1%.

EXERCISE 10
Let X be an exponential random variable such that $P(X < 1) = 0.05$. What is λ?

EXERCISE 11
Suppose that X follows $\Gamma(2, 0.5)$ ($\alpha = 2, \beta = 0.5$).

a. Find $P(X < 1)$.

b. Find $P(X < 2|X > 1)$.

EXERCISE 12
Part (a): Suppose that an average of 30 customers per hour arrive at a shop according to a Poisson process ($\lambda = \frac{1}{2}$ per minute). What is the probability that the shopkeeper will wait more than 5 minutes before two customers arrive?

Part (b): Telephone calls arrive at a switchboard at a mean rate of $\lambda = 2$ per minute according to a Poisson process. Let X denote the waiting time in minutes until the fifth call arrives. Find the pdf of X, and the mean and variance of X.