

University of California, Los Angeles
Department of Statistics

Statistics 100A

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Homework 4

EXERCISE 1

The life of a certain type of automobile tire is normally distributed with mean 34000 miles and standard deviation 4000 miles.

- a. What is the probability that such a tire lasts over 40000 miles?
- b. Given that one of these tires has survived more than 30000 miles, what is the conditional probability that it survives for more than another 10000 miles?
- c. Find the probability that 2 out of 5 randomly selected tires will have lifetime 1.96 standard deviations above the mean.

EXERCISE 2

Let X be a random variable with probability density function

$$f(x) = \begin{cases} c(1-x^2)^2 & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

- a. What is the value of c ?
- b. Find the cumulative distribution function of X ?

EXERCISE 3

A filling station is supplied with gasoline once a week. Its weekly volume of sales in thousands of gallons is a random variable with probability density function $f(x) = c(1-x)^4$, for $0 < x < 1$, and $f(x) = 0$ otherwise.

- a. What is the value of c ?
- b. What need the capacity of the tank be so that the probability of the supply's being exhausted in a given week is 1%?

EXERCISE 4

The length of the tails of a certain race of dogs follows the normal distribution with mean μ and standard deviation σ . It is known that 5% of the tails is longer than 12 inches. It is also known that 2.5% of the tails is shorter than 7 inches.

- a. Find μ and σ .
- b. Suppose that a dog is randomly selected. What is the probability that the length of its tail will be longer than 11 inches?

EXERCISE 5

Let $F(x) = 1 - \exp(-\alpha x^\beta)$ for $x \geq 0$, $\alpha > 0$, $\beta > 0$, and $F(x) = 0$ for $x < 0$. Show that F is a cumulative distribution function, and find the corresponding density.

EXERCISE 6

Let $f(x) = \frac{1+\alpha x}{2}$ for $-1 \leq x \leq 1$ and $f(x) = 0$ otherwise, where $-1 \leq \alpha \leq 1$. Show that f is a probability density function, and find the corresponding cumulative distribution function. Find the quartiles and the median of the distribution in terms of α .

EXERCISE 7

Suppose that X has the probability density function $f(x) = cx^2$ for $0 \leq x \leq 1$ and $f(x) = 0$ otherwise.

- a. Find the constant c .
- b. Find the cumulative distribution function.
- c. What is $P(0.1 \leq X \leq 0.5)$?

EXERCISE 8

Find the lower and upper quartiles (25_{th} and 75_{th} percentiles) of the exponential distribution.

EXERCISE 9

Suppose that the lifetime of an electronic component follows an exponential distribution with parameter $\lambda = 0.1$.

- a. Find the probability that the lifetime is less than 10.
- b. Find the probability that the lifetime is between 5 and 15.
- c. Find t such that the lifetime is greater than t is 1%.

EXERCISE 10

Let X be an exponential random variable such that $P(X < 1) = 0.05$. What is λ ?

EXERCISE 11

Suppose that X follows $\Gamma(2, 0.5)$ ($\alpha = 2$, $\beta = 0.5$).

- a. Find $P(X < 1)$.
- b. Find $P(X < 2 | X > 1)$.

EXERCISE 12

Part (a): Suppose that an average of 30 customers per hour arrive at a shop according to a Poisson process ($\lambda = \frac{1}{2}$ per minute). What is the probability that the shopkeeper will wait more than 5 minutes before two customers arrive?

Part (b): Telephone calls arrive at a switchboard at a mean rate of $\lambda = 2$ per minute according to a Poisson process. Let X denote the waiting time in minutes until the fifth call arrives. Find the pdf of X , and the mean and variance of X .