

University of California, Los Angeles
Department of Statistics

Statistics 100A

Instructor: Nicolas Christou

Homework 6

EXERCISE 1

Show that $Var(X - Y) = Var(X) + Var(Y) - 2Cov(X, Y)$.

EXERCISE 2

If X and Y are independent variables with equal variances find $Cov(X + Y, X - Y)$.

EXERCISE 3

If $U = a + bX$ and $V = c + dY$. show that $|\rho_{UV}| = |\rho_{XY}|$.

EXERCISE 4

Let U and V be independent random variables with means μ and variances σ^2 . Let $Z = \alpha U + V\sqrt{1 - \alpha^2}$. Find $E(Z)$ and ρ_{UZ} .

EXERCISE 5

Suppose that X and Y are two independent measurements. Also it is given that $E(X) = E(Y) = \mu$, but σ_X and σ_Y are unequal. The two measurements are combined by means of a weighted average to give

$$Z = \alpha X + (1 - \alpha)Y$$

where α is a scalar and $0 \leq \alpha \leq 1$.

- a. Show that $E(Z) = \mu$.
- b. Find α in terms of σ_X and σ_Y to minimize $Var(Z)$.
- c. Under what circumstances is it better to use the average $\frac{X+Y}{2}$ than either X or Y alone to estimate μ . Note: $X, Y, \frac{X+Y}{2}$ all have the same expected value (μ). Therefore we would prefer $\frac{X+Y}{2}$ if its variance is smaller than the variance of X and the variance of Y .

EXERCISE 6

Suppose that X_i , where $i = 1, \dots, n$, are independent random variables with $E(X_i) = \mu$ and $Var(X_i) = \sigma^2$. Let $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$. Show that $E(\bar{X}) = \mu$ and $Var(\bar{X}) = \frac{\sigma^2}{n}$.

EXERCISE 7

Let $T = \sum_{k=1}^n kX_k$, where X_k are independent random variables with mean μ and variance σ^2 . Find $E(T)$ and $Var(T)$.

EXERCISE 8

Let X and Y have the following joint probability density function:

$$f_{XY}(x, y) = \frac{6}{7}(x + y)^2, \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1$$

- a. Find the covariance and correlation of X and Y .
- b. Find $E(Y|X = x)$ for $0 \leq x \leq 1$.

EXERCISE 9

Three stocks A, B, C have the following expected (mean) returns and standard deviations:

	μ	σ
A	20%	8%
B	10%	4%
C	15%	6%

Also, the correlation coefficients are: $\rho_{AB} = 0.5$, $\rho_{AC} = 0.2$, and $\rho_{BC} = -1$. You have \$100000 to invest.

- If you invest \$75000 in stock A and \$25000 in stock B what will be the expected (mean) return of this portfolio?
- What is the risk (variance) of the portfolio of part (a)?
- You are risk averse. Which portfolio has less risk:
 - The portfolio of part (a)?
 - A portfolio in which you invest \$50000 in stock B , and \$50000 in stock C ?
- You equally allocate your funds into the three stocks A, B, C . What is the risk of this portfolio?

Exercise 10

Answer the following questions:

- If Z is a standard normal variable, what is $Cov(Z, Z^2)$?
- If Z is a standard normal variable and if Y is defined by $Y = a + bZ + cZ^2$, where a, b , and c are constants show that $\rho(Y, Z) = \frac{b}{\sqrt{b^2 + 2c^2}}$.