University of California, Los Angeles
Department of Statistics

Statistics 100A **Instructor: Nicolas Christou**

Homework 7

EXERCISE 1
Show that $Var(X-Y) = Var(X) + Var(Y) - 2Cov(X,Y)$.

EXERCISE 2
If X and Y are independent variables with equal variances find $Cov(X+Y, X-Y)$.

EXERCISE 3
If $U = a + bX$ and $V = c + dY$. show that $|\rho_{UV}| = |\rho_{XY}|$.

EXERCISE 4
Let U and V be independent random variables with means μ and variances σ^2. Let $Z = \alpha U + V\sqrt{1-\alpha^2}$. Find $E(Z)$ and ρ_{UZ}.

EXERCISE 5
Suppose that X and Y are two independent measurements. Also it is given that $E(X) = E(Y) = \mu$, but σ_X and σ_Y are unequal. The two measurements are combined by means of a weighted average to give

$$Z = \alpha X + (1-\alpha)Y$$

where α is a scalar and $0 \leq \alpha \leq 1$.

a. Show that $E(Z) = \mu$.

b. Find α in terms of σ_X and σ_Y to minimize $Var(Z)$.

c. Under what circumstances is it better to use the average $\frac{X+Y}{2}$ than either X or Y alone to estimate μ. Note: $X, Y, \frac{X+Y}{2}$ all have the same expected value (μ). Therefore we would prefer $\frac{X+Y}{2}$ if its variance is smaller than the variance of X and the variance of Y.

EXERCISE 6
Suppose that X_i, where $i = 1, \cdots, n$, are independent random variables with $E(X_i) = \mu$ and $Var(X_i) = \sigma^2$. Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Show that $E(\bar{X}) = \mu$ and $Var(\bar{X}) = \frac{\sigma^2}{n}$.

EXERCISE 7
Let $T = \sum_{k=1}^{n} kX_k$, where X_k are independent random variables with mean μ and variance σ^2. Find $E(T)$ and $Var(T)$.

EXERCISE 8
Let X and Y have the following joint probability density function:

$$f_{XY}(x,y) = \frac{6}{7}(x+y)^2, \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1$$

a. Find the covariance and correlation of X and Y.

b. Find $E(Y|X=x)$ for $0 \leq x \leq 1$.

EXERCISE 9

Three stocks A, B, C have the following expected (mean) returns and standard deviations:

	μ	σ
A	20%	8%
B	10%	4%
C	15%	6%

Also, the correlation coefficients are: $\rho_{AB} = 0.5, \rho_{AC} = 0.2$, and $\rho_{BC} = -1$. You have \$100000 to invest.

a. If you invest \$75000 in stock A and \$25000 in stock B what will be the expected (mean) return of this portfolio?

b. What is the risk (variance) of the portfolio of part (a)?

c. You are risk averse. Which portfolio has less risk:

   i. The portfolio of part (a)?

   ii. A portfolio in which you invest \$50000 in stock B, and \$50000 in stock C?

d. You equally allocate your funds into the three stocks A, B, C. What is the risk of this portfolio?

Exercise 10

Answer the following questions:

a. If Z is a standard normal variable, what is $Cov(Z, Z^2)$?

b. If Z is a standard normal variable and if Y is defined by $Y = a + bZ + cZ^2$, where a, b, and c are constants show that $\rho(Y, Z) = \frac{b}{\sqrt{b^2+2c^2}}$.