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# Department of Statistics

## Statistics 100A — Instructor: Nicolas Christou

## Examples - solutions

**Example 1:**

$$E(N) = E(C\pi r^2) = C\pi E r^2 = C\pi(\sigma^2 + \mu^2).$$

The mean and variance of the distribution of $r$ are: $E(r) = 23.5$, $var(r) = 1.54$. Therefore,

$$E(N) = 8\pi(1.54 + 23.4^2) = 13800.39 \approx 13801.$$

Or simply compute $E(N)$ as:

$$E(N) = E(C\pi r^2) = C\pi \sum_r r^2 P(r) = 8\pi \left[21^2(0.05) + 22^2(0.20) + \ldots + 26^2(0.05)\right] = 13800.39 \approx 13801.$$

**Example 2:**
It is given that $P(X = i) = cP(X = i - 1)$ for $i = 1, 2$. Let P(X=0)=p.
$P(X = 1) = cP(X = 0) = cp$, and $P(X = 2) = cP(X = 1) = c^2 p$. Also, $P(X = 0) + P(X = 1) + P(X = 2) = 1$.
Or $p + cp + c^2 p = 1 \Rightarrow p = \frac{1}{1+c+c^2}$. The expected value of $X$ is:

$$E(X) = 0(p) + 1(cp) + 2(c^2 p) = \frac{c}{1+c+c^2} + \frac{2c^2}{1+c+c^2} = \frac{c(1+2c)}{1+c+c^2}.$$

**Example 3:**
There are 8 white, 4 black and 2 orange balls. Two balls are selected without replacement. For each black we win \$2, for each white we lose \$1. We neither win nor we lose anything if we select an orange ball.

a.

| Color | $X$(\$) | $P(X)$ |
|---|---|---|
| $WW$ | $-2$ | $\frac{8}{14}\frac{7}{13} = \frac{56}{182}$ |
| $WO$ or $OW$ | $-1$ | $2\frac{8}{14}\frac{2}{13} = \frac{32}{182}$ |
| $OO$ | $0$ | $\frac{2}{14}\frac{1}{13} = \frac{2}{182}$ |
| $BW$ or $WB$ | $1$ | $2\frac{4}{14}\frac{8}{13} = \frac{64}{182}$ |
| $BO$ or $OB$ | $2$ | $2\frac{4}{14}\frac{2}{13} = \frac{16}{182}$ |
| $BB$ | $4$ | $\frac{4}{14}\frac{3}{13} = \frac{12}{182}$ |

b. Expected value of our winnnings:
$E(X) = -2\frac{56}{182} + -1\frac{32}{182} + 0\frac{2}{182} + 1\frac{64}{182} + 2\frac{16}{182} + 4\frac{12}{182} = 0.$

c. Standard deviation of our winnings:
$\sigma^2 = (-2)^2\frac{56}{182} + (-1)^2\frac{32}{182} + 0^2\frac{2}{182} + 1^2\frac{64}{182} + 2^2\frac{16}{182} + 4^2\frac{12}{182} - 0^2 = \frac{576}{182}$. Therefore the standard deviation is $\sigma = \sqrt{\frac{576}{182}} = 1.78.$

d.
$$P(X = -2|X < 0) = \frac{P(X = -2 \cap X < 0)}{P(X < 0)} = \frac{P(X < 0|X = -2)P(X = -2)}{P(X < 0)} = \frac{1\,\frac{56}{182}}{\frac{56}{182} + \frac{32}{182}} = \frac{56}{88} = 0.64.$$

**Example 4:**
Let $X$ be the number of tests needed for each group of 10 people. Then, if nobody has the disease 1 test is enough. But if the test is positive then there will be 11 test $(1 + 10)$. The probability distribution of $X$ is:

| $X$ | $P(X)$ |
|---|---|
| 1 | $\binom{10}{0} 0.10^0 0.90^{10} = 0.90^{10}$ |
| 11 | $1 - \binom{10}{0} 0.10^0 0.90^{10} = 1 - 0.90^{10}$ |

Therefore the expected number of tests is:
$E(X) = 1(0.90)^{10} + 11(1 - 0.90^{10}) = 7.51.$

**Example 5:**
Using example 4 when $n = 2, 4, 5, 20$ we get the following: When $n = 2$, $E(X) = 1.38$. The total number of tests for the 100 people is $1.38(50) = 69$.
When $n = 4$, $E(X) = 2.38$. The total number of tests for the 100 people is $2.38(25) = 59.5$.
When $n = 5$, $E(X) = 3.05$. The total number of tests for the 100 people is $3.05(20) = 61$.
When $n = 20$, $E(X) = 18.57$. The total number of tests for the 100 people is $18.57(5) = 92.9$.
Therefore to minimize the number of tests we must place them in groups of 4.

**Example 6:**
The horse will win both races with probability 0.06, one race with probability 0.38, and no race with probability 0.56. The probability distribution of the profit $X$ will be:

| $X$ | $P(X)$ |
|---|---|
| 80000 | 0.06 |
| 30000 | 0.38 |
| -10000 | 0.56 |

The expected value and standard deviation of the profit are:

$$E(X) = 80000(0.06) + 30000(0.38) - 10000(0.56) = 10600.$$

$$SD(X) = \sqrt{80000^2(0.06) + 30000^2(0.38) + 10000^2(0.56) - 10600^2} = 25877.40.$$