PAGES 12–13 : REVIEW PROBLEMS (HANDOUT # 6)

PROBLEM 1 : START FROM RHS :

(a). $$\frac{p(n-x)}{(x+1)(1-p)} P(X=x) = \frac{p(n-x)}{(x+1)(1-p)} \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \frac{p(n-x)}{(x+1)(1-p)} \frac{n!}{(n-x)!\,x!} p^x (1-p)^{n-x}$$

$$= \frac{(n-x)\,n!}{(x+1)!\,(n-x)\,(n-x-1)!} \frac{p}{1-p} p^x (1-p)^{n-x}$$

$$= \frac{n!}{(n-x-1)!\,(x+1)!} p^{x+1} (1-p)^{n-x-1}$$

$$= \binom{n}{x+1} p^{x+1} (1-p)^{n-x-1} = P(X=x+1).$$

(b). $$P(X=1) = \frac{0.25\,(8-0)}{(0+1)(1-0.25)} 0.1001 \Rightarrow P(X=1) = 0.2669.$$

$$P(X=2) = \frac{0.25\,(8-1)}{(1+1)(1-0.25)} 0.2669 \Rightarrow P(X=2) = 0.3114.$$

PROBLEM 6:

(a). GEOMETRIC. $P(X=1) = 0.4$

$P(X=2) = (0.6)(0.4) = 0.24$

$P(X=3) = (0.6^2)(0.4) = 0.144$

(b). NEGATIVE BINOMIAL.

$P(X=4) = \binom{4-1}{2-1} 0.4^2\, 0.6^2 = 0.1728.$

PROBLEM 7: $X \sim \text{Poisson}(2)$

$EX = \mu = 2$

$VAR(X) = \sigma^2 = 2$

$Y = 50 - 2X - X^2$

$EY = E(50 - 2X - X^2)$

$= 50 - 2EX - EX^2$

$= 50 - 2\mu - (\sigma^2 + \mu^2)$

$= 50 - 2(2) - (2 + 2^2) \Rightarrow \underline{EY = 40}.$

PROBLEM 8 :

$$P(Y=x) = P(X=x \mid X>0) = \frac{P(X=x \cap X>0)}{P(X>0)}$$

$$= \frac{P(X>0 \mid X=x)\,P(X=x)}{P(X>0)} = \frac{P(X=x)}{P(X>0)}$$

$$\therefore\ P(Y=x) = \frac{P(X=x)}{P(X>0)}$$

$$\left.\begin{aligned} P(Y=1) &= \frac{P(X=1)}{P(X>0)} \\ P(Y=2) &= \frac{P(X=2)}{P(X>0)} \end{aligned}\right\} \Longrightarrow$$

Y	$P(y)$
1	$\frac{P(X=1)}{P(X>0)}$
2	$\frac{P(X=2)}{P(X>0)}$
3	$\frac{P(X=3)}{P(X>0)}$
$\vdots$	$\vdots$

THEREFORE, $$EY = \sum_{y=1}^{\infty} y\,P(y)$$

$$= \frac{\sum_{x=0}^{\infty} x\,P(X=x)}{P(X>0)} = \frac{EX}{1-P(X=0)}$$

$$= \frac{\lambda}{1-\frac{\lambda^0 e^{-\lambda}}{0!}} \Rightarrow \boxed{EY = \frac{\lambda}{1-e^{-\lambda}}}$$