University of California, Los Angeles
Department of Statistics

Statistics 100A

Instructor: Nicolas Christou

Exam 1
21 October 2011

Name: SOLUTIONS

Problem 1 **(20 points)**
Answer the following questions:

a. Let X follow the Poisson distribution with parameter λ. Show that

$$EX^n = \lambda E(X+1)^{n-1}. \quad \textit{Hint: } \text{Write } X^n = XX^{n-1} \text{ and then find } EXX^{n-1}.$$

Then use this result to find EX^3.

$$EX^n = EXX^{n-1} = \sum_{x=1}^{\infty} x x^{n-1} \frac{\lambda^x e^{-\lambda}}{x!} = \sum_{x=1}^{\infty} \frac{x^{n-1} \lambda^x e^{-\lambda}}{(x-1)!}$$

$$= \lambda \sum_{x=1}^{\infty} \frac{x^{n-1} \lambda^{x-1} e^{-\lambda}}{(x-1)!} \qquad \text{LET } Y = X-1 \Rightarrow X = Y+1$$

$$= \lambda \sum_{y=0}^{\infty} (y+1)^{n-1} \frac{\lambda^y e^{-\lambda}}{y!} = \lambda E(X+1)^{n-1}$$

$$EX^3 = \lambda E(X+1)^2 = \lambda E(X^2+2X+1) = \lambda[EX^2+2EX+1]$$

$$= \lambda(\sigma^2+\mu^2+2\mu+1) = \lambda[\lambda+\lambda^2+2\lambda+1]$$

$$= \lambda(3\lambda+\lambda^2+1).$$

b. Suppose there are n people in a room. Compute or explain how to find the minimum value of n so that the probability that no 4 of them have the same birthday is less than $\frac{1}{2}$ using the Poisson distribution.

$$P(\text{ALL FOUR HAVE SAME BIRTHDAY}) = \frac{1}{365^3}$$

$$\lambda = \binom{n}{4} \frac{1}{365^3}$$

WE WANT: $P\left(\text{NO 4 HAVE SAME BIRTHDAY}\right) = P(X=0) < \frac{1}{2}$

OR $\frac{\lambda^0 e^{-\lambda}}{0!} < \frac{1}{2} \Rightarrow e^{-\binom{n}{4}\frac{1}{365^3}} < \frac{1}{2}$

$$\Rightarrow e^{\binom{n}{4}\frac{1}{365^3}} > 2 \Rightarrow \binom{n}{4}\frac{1}{365^3} > \ln 2 \ldots$$

Problem 2 **(20 points)**
Answer the following questions:

a. Compute the expected value and standard deviation of the *minimum* number when two dice are rolled. You will have to find the distribution of the minimum (X) first (i.e. complete the table below).

X	$P(X)$
1	1/36
2	3/36
3	5/36
4	7/36
5	9/36
6	11/36

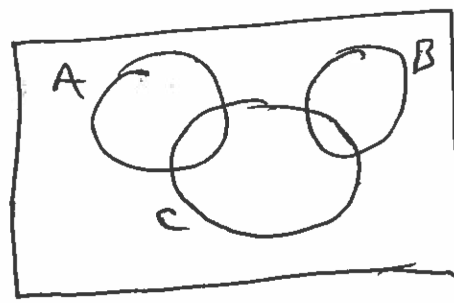
$$EX = \sum x P(x) = 1 \cdot \frac{1}{36} + \cdots + 6 \, \frac{11}{36} = 4.47$$

$$VAR(X) = \sum x^2 P(x) - \mu^2 = 1^2 \frac{1}{36} + \cdots + 6^2 \frac{11}{36} - (4.47)^2$$

$$VAR(x) = 1.99$$

$$SD(x) = \sqrt{1.99} = 1.4\cancel{107}.$$

b. Suppose that A, B, and C are three events such that
A, B are disjoint,
A, C are independent, and
B, C are independent.
Suppose also that $4P(A) = 2P(B) = P(C)$, and $P(A \cup B \cup C) = 5P(A)$.
Determine the value of $P(A)$.



$$P(A \cup B \cup C) = 5P(A)$$

$$P(A) + P(B) + P(C) - \cancel{P(A \cap B)} - P(A \cap C) - P(B \cap C) + \cancel{P(A \cap B \cap C)} = 5P(A)$$

$$P(A) + 2P(A) + 4P(A) - 4P(A)P(A) - 8P(A)P(A) = 5P(A)$$

$$7P(A) - 12P^2(A) = 5P(A)$$

$$2P(A) = 12P^2(A)$$

$$\boxed{P(A) = \frac{1}{6}}$$

Problem 3 (20 points)
Answer the following questions:

a. Consider the casino roulette game. If a player bets \$1 on a single game and we let X be the casino's profit we get the following probability distribution:

X	$P(x)$
-35	$\frac{1}{38}$
1	$\frac{37}{38}$

In 500 such games, what is the *exact* probability that the casino will make more than \$200? You don't need to compute this probability, but you must write the exact expression.

LET K = # OF GAMES CASINO MUST WIN.

WE NEED: $K - 35(500-K) > 200 \Rightarrow K \geq 492$

THEN $\cancel{X}Y \sim b(500, \frac{37}{38})$

$$P(\cancel{X}Y \geq 492) = \sum_{y=492}^{500} \binom{500}{y} \frac{37}{38}^{y} \frac{1}{38}^{500-y}$$

b. The probability of receiving a one pair poker hand (for example A,A,4,5,10) is 42%. Find the probability that the 6th one pair poker hand for a player will be observed on his 10th game.

NEGATIVE BINOMIAL

$\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_}_{\text{NEED 5 OUT OF 9 TRIALS}}\ |\ \_$ 6TH SUCCESS 10TH TRIAL

$$P(X=10) = \binom{9}{5} 0.42^{5}\, 0.58^{4}\, 0.42 = \binom{9}{5} 0.42^{6}\, 0.58^{4}$$

$$P(X=10) = 0.0783$$

c. Let X follow the binomial distribution with parameters n and $p > 0$. Is it true that the mean is always larger than the standard deviation? If not, please find the condition under which this is not true.

No.! $np < \sqrt{np(1-p)}$

$n^2p^2 < np(1-p)$

$np < 1-p$

$p(n+1) < 1 \Rightarrow \boxed{p < \frac{1}{n+1}}$

$n < \frac{1-p}{p}$

Problem 4 (20 points)
Answer the following questions:

a. A company rents luxurious cars to customers for recreational purposes. Customers must rent these cars for the entire day. A decision must be made about how many of these cars the company must have. They don't want to have too many cars but at the same time they don't want to turn customers away due to lack of cars availability. Suppose that from experience we know that the number of customers follows the Poisson distribution with $\lambda = 6.1$ per day. The owner of the company would like to find the minimum number of cars so that, with probability 90% or more, all the customers will get a car. Explain how you will be able to find the minimum number of cars. You don't need to submit a final answer as this will take long, but please be very specific. One should be able to find the number of cars by reading your solution!

LET K = MINIMUM # OF CARS

WE NEED $P(X \leq K) \geq 0.90$

OR $$\sum_{X=0}^{K} \frac{6.1^X e^{-6.1}}{X!} \geq 0.90$$

SOLVE TO FIND: $K = 9$

b. Refer to part (a). Suppose that the number of customers follows the Poisson distribution with $\lambda = 6.1$ per day. The probability that at any day at least nine customers will show up is 0.16. What is the probability that it will take more than 13 days until the company observes a day with at least nine customers?

GEOMETRIC: $P(\cancel{Y} > 13) = (1-p)^{13}$

$= (1-0.16)^{13} = \underline{\underline{0.1037}}$

d. Refer to part (b). What is the probability that it will take between 6 and 16 days (including) until the company observes a day with at least nine customers?

$P(6 \leq \cancel{Y} \leq 16) = P(Y \leq 16) - P(Y \leq 5)$

$= 1 - (1-p)^{16} - [1 - (1-p)^5]$

$= 1 - 0.84^{16} - 1 + 0.84^5$

$= 0.84^5 - 0.84^{16} = \underline{\underline{0.3567}}$

Problem 5 (20 points)
Answer the following questions:

a. In a certain city, taxis are easily identified by rooftop lights. There are two taxi companies in this city, Green and Blue, with taxis colored accordingly. One night, there is a hit-and-run accident involving a taxi. The vehicle was clearly a taxi, as multiple witnesses saw the rooftop light. However, only one witness (Mr. A) claimed to be able to identify the taxi by color. Mr. A claimed that this was a Green taxi. Here are some important facts:
Mr. A was given an identification test under lighting conditions similar to those the night of the accident. When the taxi was blue, he identified it correctly 80% of the time. When the test taxi was green, he identified it correctly 80% of the time. A number of other people were given the same identification test, and they also produced the same 80% probability that Mr. A got. Also, we know that in this city, 85% of the taxis are Blue.

What is the probability that the accident was really committed by a Green taxi.

$P(\text{MR. A SAYS GREEN} \mid \text{GREEN}) = 0.80$ $\quad P(\text{MR. A SAYS GREEN} \mid \text{BLUE}) = 0.20$

$P(\text{GREEN}) = 0.15$ $\quad P(\text{BLUE}) = 0.85$

WE WANT: $P(\text{GREEN} \mid \text{MR. A SAYS GREEN})$

$$= \frac{P(\text{GREEN} \cap \text{MR. A SAYS GREEN})}{P(\text{MR. A. SAYS GREEN})}$$

$$= \frac{P(\text{MR. A SAYS GREEN} \mid \text{GREEN})\,P(\text{GREEN})}{P(\text{MR. A SAYS GREEN} \mid \text{GREEN})\,P(\text{GREEN}) + P(\text{MR. A SAYS GREEN} \mid \text{BLUE})}$$

$$= \frac{0.80 \times 0.15}{0.80 \times 0.15 + 0.20 \times 0.85} = \underline{0.438}.$$

b. A purchaser of electrical components buys them in lots of size 10. It is his policy to inspect 3 components randomly from a lot and to accept the lot only if all 3 components are nondefective. If 30 percent of the lots have 4 defective components and 70 percent have only 1, what proportion of lots does the purchaser reject? *Hint:* It will be easier first to compute the proportion of lots that the purchaser accepts.

$P(\text{ACCEPT}) = P(X=3 \cap \text{LOT A}) + P(X=3 \cap \text{LOT B})$

$$= 0.30 \times \frac{\binom{4}{0}\binom{6}{3}}{\binom{10}{3}} + 0.70\ \frac{\binom{1}{0}\binom{9}{3}}{\binom{10}{3}}$$

$10 < \begin{matrix} 4D \\ 6G \end{matrix}$ 30%

$\Rightarrow P(\text{ACCEPT}) = 0.54$

$10 < \begin{matrix} 1D \\ 9G \end{matrix}$ 70%

$\underline{P(\text{REJECT}) = 0.46}$

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Exam 1
17 July 2007

[redacted]

Name: SOLUTIONS

Problem 1 (20 points)
Part A:
Use the binomial theorem to show that:

(6) a. $\sum_{k=0}^{n} \binom{n}{k}(-1)^k = 0$.

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

Let $x=(-1)$ $y=+1$

$$0=(1-1)^n = \sum_{k=0}^{n} \binom{n}{k}(-1)^k (+1)^{n-k}$$

(6) b. $\sum_{k=0}^{n} \binom{n}{k}(a-1)^k = a^n$.

Let $x=a-1$ $y=1$

$$a^n = (a-1+1)^n = \sum_{k=0}^{n} \binom{n}{k} (a-1)^k \cdot 1^{n-k}$$

Part B:
What is the probability that among 5 people at least two have their birthday in the same month?

(8) ~~X=~~

P(at least two have the same month) = 1 − P(none of them in the same month)

$$=1-\frac{12\cdot 11\cdot 10\cdot 9\cdot 8}{12^5}$$

$$=0.618055\cdots$$

Problem 2 (20 points)

Consider a two-stage game. At the first stage, we flip a fair coin. If the coin comes up heads, we select one ball from urn 1, and if the coin comes up tails, we select a ball from urn 2. The urns have the following composition:

Urn 1: 5 green balls and 10 yellow balls.

Urn 2: 20 green balls and 10 yellow balls.

You arrived late for this game, and you missed the coin flip. However, you did get to witness the selection of a yellow ball. What is the probability that the ball came from urn 1?

$$p(U1 \mid Y) = \frac{p(Y \mid U1)\, p(U1)}{p(Y)}$$

$$= \frac{\frac{10}{15} \cdot \frac{1}{2}}{\frac{10}{15} \cdot \frac{1}{2} + \frac{10}{30} \cdot \frac{1}{2}} = \frac{20}{20+10} = \frac{2}{3}$$

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Problem 3 (20 points)

A certain company produces air filters at three different assembly plants. The first plant makes 60% of all the filters, and 1% of its filters will be returned by customers because of defects. The second plant makes 30% of all the filters, and 2% of its filters will be returned by customers because of defects. The third plant makes 10% of all the filters, and 3% of its filters will be returned by customers because of defects. In the process of packing the filters for shipment to retail outlets, the filters from the three plants are intermixed.

a. Suppose a filter is randomly selected from a particular retail outlet. What is the probability that the filter will be found defective.

(10)

$P(P1) = 0.6. \quad P(D|P_1) = 0.01$

$P(P2) = 0.3 \quad P(D|P_2) = 0.02$

$P(P_3) = 0.1 \quad P(D|P_3) = 0.03.$

$$P(D) = P(D|P_1)P(P_1) + P(D|P_2)P(P_2) + P(D|P_3)P(P_3)$$

$$= 0.6 \times 0.01 + 0.3 \times 0.02 + 0.1 \times 0.03$$

$$= 0.006 + 0.006 + 0.003$$

$$= 0.015$$

b. Suppose that a filter was returned (it was found defective). What is the probability that this filter was made at the first plant?

(10)

$$P(P_1|D) = \frac{P(D|P_1)P(P_1)}{P(D)} = \frac{0.6 \times 0.01}{0.015} = \frac{6}{15} = \frac{2}{5}$$

$$= 0.4$$

Problem 4 (20 points)

Part A:

A man buys a racehorse for \$20000, and enters it in two races. He plans to sell the horse afterward, hoping to make a profit. If the horse wins both races, its value will jump to \$100000. If it wins one of the races, it will worth \$50000. If it loses both races, it will worth only \$10000. The man believes there is a 20% chance that the horse will win the first race and a 30% chance it will win the second one.

a. Assuming that the two races are independent events, find the man's expected profit.

	X	$P(X)$
lost both	\$10000 − \$20000 = −10000	$0.8 \times 0.7 = 0.56$
win once	\$30000	$0.2\times 0.7 + 0.8 \times 0.3 = 0.38$ $0.14 + 0.24$
win both	\$80000	$0.2 \times 0.3 = 0.06$

(8)

$$\Rightarrow E(X) = -10000 \times 0.56 + 30000 \times 0.38 + 80000 \times 0.06$$
$$= -5600 \quad 11400 + 4800 \quad = 10600.$$

b. Find the standard deviation of the man's profit.

$$E(X^2) = (10000)^2 \times 0.56 + (30000)^2 \times 0.38 + (80000)^2 \times 0.06$$
$$= 0.56 \times (10000)^2 + 3.42 \times (10000)^2 + 3.84 \times (10000)^2$$
$$= 7.82 \times (10000)^2$$

(6)

$$Var(X) = EX^2 - (EX)^2 = 7.82 \times (10000)^2 - 1.06^2 \times (10000)^2$$
$$= 6.6964 \times (10000)^2 \ \cancel{\# \ 25877 \times 10000^2}$$

$$\sigma = \sqrt{Var\, X} \qquad = 25877$$

Part B:

Suppose that X takes on one of the values 0, 1, 2. If for some constant c, $P(X = i) = cP(X = i - 1), i = 1, 2$ find $E(X)$ in terms of c.

$P(X=1) = c \cdot P(0)$

$P(X=2) = c \cdot P(1)$

X	$P(X)$
0	p
1	cp
2	c^2p

$$EX = 0 \cdot p + 1 \cdot cp + 2 \cdot c^2 p$$
$$= cp + 2c^2p$$
$$= (c + 2c^2) \frac{1}{1+c+c^2} = \frac{c(1+2c)}{1+c+c^2}$$

(6)

$$\Rightarrow p + cp + c^2p = 1$$

$\Rightarrow$ ...

Problem 5 (20 points)

Let X be a discrete random variable which has the following probability distribution.

X	$P(X)$
0	0.2
1	0.3
2	0.4
3	0.1

a. Compute $E(4+X)$.

$E(X) = 0 \times 0.2 + 1 \times 0.3 + 2 \times 0.4 + 3 \times 0.1$

$= 0.3 + 0.8 + 0.3 = 1.4.$

$E(4+X) = 4 + EX = 4 + 1.4 = 5.4$

(5)

b. Compute $\mathrm{Var}(X)$.

$E(X^2) = 0^2 \times 0.2 + 1^2 \times 0.3 + 2^2 \times 0.4 + 3^2 \times 0.1$

$= 0 + 0.3 + 1.6 + 0.9 = 2.8$

$\mathrm{Var}\,X = EX^2 - (EX)^2 = 2.8 - 1.4^2 = 0.84$

(5)

c. Compute $E(X^2 - 2X + 1)$.

$E(X^2 - 2X + 1) = EX^2 - 2EX + 1 = 2.8 - 2 \times 1.4 + 1$

$= 2.8 - 2.8 + 1$

$= 1$

(5)

d. Let $Y = 0.4X - 2$. Compute $\mathrm{Var}(Y)$.

$\mathrm{Var}(Y) = \mathrm{Var}(0.4X - 2) = 0.4^2 \cdot \mathrm{Var}\,X = 0.16 \times 0.84$

$= 0.1344$

(5)

University of California, Los Angeles
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Exam 1
15 July 2008

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Name: SOLUTIONS

Problem 1 (20 points)
Answer the following questions:

(6)

a. What is the probability that you and the person sitting next to you have the same birthday?

$$1 - \frac{365 \cdot 364}{365^2} = \frac{1}{365}$$

(6)

b. Two dice are rolled and the sum of the two numbers is observed. What is the probability that the sum of 8 will appear before the sum of 7?

$$P(8 \text{ before } 7) = P(8 \mid 7 \text{ or } 8) = \frac{P(8)}{P(7 \text{ or } 8)}$$

$$= \frac{5/36}{6/36 + 5/36} = \frac{5}{11} = \underline{0.4545}.$$

c. Two events A, B are independent. Show that A and B' are also independent.

(8)

$$P(A \cap B') = P(A) - P(A \cap B)$$
$$= P(A) - P(A) \cdot P(B)$$
$$= P(A)[1 - P(B)]$$

$$\therefore P(A \cap B') = P(A) \cdot P(B')$$

INDEPENDENT

Problem 2 (20 points)
Answer the following questions:

(5)

a. There are five card decks A, B, C, D, E (each deck has a different color). The cards are shuffled together in one pile of 254 cards. From this pile, 10 cards are selected without replacement. What is the probability that in these 10 cards there will be exactly 2 cards from each one of the card decks?

$$\frac{\binom{52}{2}\binom{52}{2}\binom{52}{2}\binom{52}{2}\binom{52}{2}}{\binom{254}{10}} = \underline{\underline{0.0159}}.$$

(5)

b. The probability of winning a roulette game is $\frac{4}{38}$. What is the probability that a player will win at least once in 10 games?

$$1 - \left(\frac{34}{38}\right)^{10} = \underline{\underline{0.6712}}.$$

(5)

c. The probability of winning the game of craps is 0.49. A player repeatedly plays the game until he wins. What is the probability that his first win will occur on the 10_{th} game?

$$P(X=10) = 0.51^{9}\, 0.49 = \underline{\underline{0.0011}}$$

(5)

d. Refer to part (c). What is the probability that his first win will occur *after* the 10_{th} game?

$$P(X>10) = 0.51^{10} = \underline{\underline{0.0012}}.$$

Problem 3 **(20 points)**

You ask your neighbor to water your plant while you are on vacation. Without water it will die with probability 0.80. With water it will die with probability 0.15. You are 0.90 percent certain that your neighbor will remember to water the plant.

(8) a. What is the probability that the plant will be alive when you return?

$P(D|W') = 0.80$ $\quad P(D|W) = 0.15$ $\quad P(W) = 0.90$

$P(A|W') = 0.20$ $\quad P(A|W) = 0.85$ $\quad P(W') = 0.10$

$$P(A) = P(A \cap W) + P(A \cap W')$$
$$= P(A/W)P(W) + P(A/W')P(W')$$
$$= (0.85)(0.90) + (0.20)(0.10) = \underline{\underline{0.785}}$$

(4) b. What is the probability that the plant will not be alive when you return?

$$P(A') = 1 - 0.785 = \underline{\underline{0.215}}.$$

(8) c. If the plant is dead, what is the probability your neighbor forgot to water it?

$$P(W'|D) = \frac{P(D \cap W')}{P(D)} = \frac{P(D|W')P(W')}{P(D)}$$
$$= \frac{(0.80)(0.10)}{0.215} = \underline{\underline{0.3721}}.$$

Problem 4 (20 points)
Answer the following questions:

a. The probability of receiving a one-pair poker hand is 0.42. What is the probability that a player will receive exactly 3 one-pair hands in 10 games.

(5)

$$P(X=3) = \binom{10}{3} 0.42^3 \, 0.58^7 = \underline{\underline{0.1963}}.$$

b. Compute the expected value and standard deviation of the *minimum* number when two dice are rolled. You will have to find the distribution of the minimum (X) first (i.e. complete the table below).

(5)

X	$P(X)$
1	11/36
2	9/36
3	7/36
4	5/36
5	3/36
6	1/36

$$EX = \cdots\cdots = \underline{\underline{2.52778}}.$$

$$SD(X) = \cdots\cdots = \underline{\underline{1.40408}}.$$

c. A random variable X has $E(X) = 5$ and $SD(X) = 2$. Find $E(X^2 + 2X)$.

(5)

$$E(X^2+2X) = EX^2 + 2EX$$
$$= \sigma^2 + \mu^2 + 2\mu = 4 + 25 + 2(5)$$
$$\therefore \; E(X^2+2X) = \underline{\underline{39}}.$$

d. Two players A, B compete for a prize by rolling two dice until one scores a sum of 5. What is the probability that player A wins? Note: player A begins the game.

5)

$$P(A) = P\left(\begin{matrix}A \text{ ON} \\ \text{HIS 1ST}\end{matrix}\right) + P\left(\begin{matrix}A \text{ ON} \\ \text{HIS 2ND}\end{matrix}\right) + P\left(\begin{matrix}A \text{ ON} \\ \text{HIS 3RD}\end{matrix}\right) + \cdots$$

$$= \underset{\underline{A}}{\frac{4}{36}} + \underset{\underline{A'}\,\underline{B'}\,\underline{A}}{\left(\frac{32}{36}\right)^2 \frac{4}{36}} + \underset{\underline{A'}\,\underline{B'}\,\underline{A'}\,\underline{B'}\,\underline{A}}{\left(\frac{32}{36}\right)^4 \frac{4}{36}} + \cdots$$

$$= \frac{4/36}{1-\left(\frac{32}{36}\right)^2} = \frac{36}{68} = \underline{\underline{0.5294}}.$$

Problem 5 (20 points)
The probability that a consumer will return a particular electronic device is 5%. Suppose a store has sold 100 of these electronic components in one week.

(5) a. What is the expected number and standard deviation of the number of returned electronic devices for repair?

$X \sim bin(100, 0.05)$

$EX = np = 100(0.05) = 5$

$SD(X) = \sqrt{np(1-p)} = \sqrt{100(0.05)(0.95)} = \underline{2.18}$

(5) b. Write an expression of the probability that at least 7 electronic devices will return for repair.

$P(X \geq 7) = \sum_{x=7}^{100} \binom{100}{x} 0.05^{x} \, 0.95^{100-x}$

OR $P(X \geq 7) = 1 - P(X \leq 6) = 1 - \sum_{x=0}^{6} \binom{100}{x} 0.05^{x} \, 0.95^{100-x}$

(5) c. The total cost for repairing the defective electronic devices (among the 100 sold) is given by $C = X^2 - 3X + 50$, where X is the number of defective electronic devices among the 100 sold. Find the expected cost.

$E(C) = E(X^2 - 3X + 50) = EX^2 - 3EX + 50$

$= \sigma^2 + \mu^2 - 3\mu + 50 = 100(0.05)(0.95) + [100(0.05)]^2 - 3 \cdot 100(0.05) + 50$

$\Rightarrow \underline{E(C) = 64.75}$

(5) d. Another store has sold 50 such electronic components. What is the combined expected cost for the two stores? Assume that the second store has the same cost function as in part (c).

$E(C_1 + C_2) = E(C_1) + E(C_2)$

$E(C_2) = 50(0.05)(0.95) + [50(0.05)]^2 - 3 \cdot 50 \cdot (0.05) + 50 = 51.13$

$E(C_1 + C_2) = 64.75 + 51.13 = \underline{115.88}$