University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Exam 1
26 April 2013

Name: ____________________

Problem 1 (25 points)
Answer the following questions:

a. Suppose $\bar{X}$ is unbiased estimator of μ_X. Is $\bar{X}^2$ an unbiased estimator of μ_X^2?

b. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with $X_i \sim N(0, \sigma)$. Find $E\frac{X_i^2}{\sigma^2}$.

c. Refer to part (b). What is the distribution of $\frac{X_1}{\sqrt{\frac{\sum_{i=2}^n X_i^2}{n-1}}}$? Please show all your work.

d. Suppose the logarithm of a stock price in six months will follow the normal distribution with mean \$50 and standard deviation \$20. Find the probability that the stock price in six months will exceed \$79. Note: It is given that $ln(X) \sim N(50, 20)$.

Problem 2 (25 points)
Answer the following questions:

a. Let $X \sim \Gamma(\alpha, \beta)$. Show that $\frac{2}{\beta} X \sim \chi^2_{2\alpha}$.

b. Refer to part (a). Suppose X_1, X_2, X_3, X_4 are i.i.d. random variables with $X_i \sim \Gamma(3, 4)$. Find c such that $P(\sum_{i=1}^{4} X_i > c) = 0.05$.

c. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with $X_i \sim N(0, \sigma)$. Find $\hat{\sigma^2}$, the mle of σ^2.

d. Refer to part(c). Is $\hat{\sigma^2}$ MVUE of σ^2?

Problem 3 (25 points)
Answer the following questions:

a. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with $X_i \sim N(\theta, 1)$. Let $\hat{\theta}$ be the mle of θ. Is $\hat{\theta}$ MVUE of θ?

b. If X has a binomial distribution with parameters n and p, then $\hat{p}_1 = \frac{X}{n}$ is an unbiased estimator of p. Another estimator of p is $\hat{p}_2 = \frac{X+1}{n+2}$. Find the bias of $\hat{p}_2$.

c. Refer to part (b). Find $MSE(\hat{p}_1)$ and $MSE(\hat{p}_2)$.

d. Assume that you have a sample mean $\bar{X}$ and a sample variance S^2 based on a random sample $X_1, X_2, \ldots, X_n$ from a normal distribution. Suppose you want to predict a new value X_p. Find the distribution of $X_p - \bar{X}$ and use it together with the distribution of S^2 to form a ratio that follows the t distribution. Note: X_p is independent of $X_1, X_2, \ldots, X_n$.

Problem 4 (25 points)

Consider k independent random variables that have normal distributions with unknown means $\mu_1, \mu_2, \ldots, \mu_k$, respectively, and unknown but common variance σ^2. Random samples of size $n_i, i = 1, 2, \ldots, n_k$ are selected from these k populations. Answer the following questions:

a. Write the likelihood and log-likelihood function that involves all the information above.

b. Use the method of maximum likelihood to find estimates of $\mu_i, i = 1, 2, \ldots, k$ and σ^2. Denote these estimates with $\hat{\mu}_i, i = 1, 2, \ldots, k$ and $\hat{\sigma^2}$.

c. Is $\hat{\sigma^2}$ unbiased estimator of σ^2? If not, give the unbiased estimator of σ^2 and denote it with S_k^2.

d. Find the distribution associated with S_k^2.