University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Exam 1
03 February 2012

Name: ____________________________

Problem 1 (25 points)
Answer the following questions:

a. The pdf of a random variable X is $f(x) = \frac{1}{4}xe^{-\frac{x}{2}}$. What is the distribution of X.

b. Let $X \sim \Gamma(\frac{n}{2}, \beta)$. Find the distribution of $Y = \frac{2X}{\beta}$.

c. Let $X \sim \Gamma(\alpha, \beta)$. A random sample $X_1, X_2, \ldots, X_n$ is selected from this distribution. Suppose that α is known and $\hat{\beta} = \frac{\bar{X}}{\alpha}$ is an estimator of β. Is $\hat{\beta}$ MVUE of β?

Problem 2 (25 points)
Let X_1, X_2, X_3, X_4 be iid random variables each one with probability $N(10, 4\sigma)$.
Let Y_1, Y_2, Y_3 be iid random variables each one with probability $N(5, 3\sigma)$.
The two samples are independent. Also, let $\bar{X}, \bar{Y}, S_X^2, S_Y^2$ be the corresponding sample means and sample variances of the two samples. Answer the following questions:

a. What is the distribution of $\bar{X} - \bar{Y}$?

b. What is the distribution of $\frac{3S_X^2}{16\sigma^2} + \frac{2S_Y^2}{9\sigma^2}$?

c. Use parts (a) and (b) to form a ratio that follows the t distribution. What are the degrees of freedom?

d. Using the ratio of part (c) find k such that:

$$P\left(\frac{\bar{X} - \bar{Y} - 5}{\sqrt{\frac{3S_X^2}{16} + \frac{2S_Y^2}{9}}} > k\right) = 0.05.$$

Problem 3 (25 points)
Let $X_1, X_2, \cdots, X_n$ denote a random sample from a normal distribution with mean zero and unknown variance σ^2.

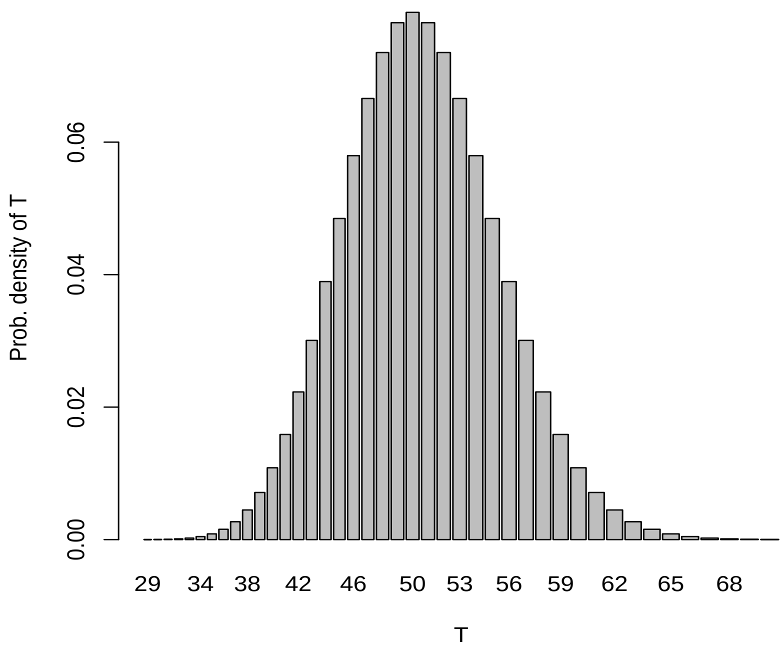
a. Let $\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} x_i^2}{n}$ be an estimator of σ^2. Is it unbiased?

b. Find the variance of $\hat{\sigma}^2$. Is it consistent?

c. Show that the variance of the estimate of part (a) is equal to the Cramér-Rao lower bound.

d. Another estimator for σ^2 is $S^2 = \frac{\sum_{i=1}^{n} x_i^2}{n-1}$. Find the MSE of S^2 and compare it to the MSE of $\hat{\sigma}^2$.

Problem 4 (25 points)
Part A:
A population has mean $\mu = 1.25$ and standard deviation $\sigma = 2$. Repeated samples each one of size $n = 40$ are selected from this population. It is claimed that the histogram below represents the distribution of the total (T) of these 40 observations. Clearly explain if there is anything wrong with this histogram.



Part B:
Let $X_1, X_2, \ldots, X_{19}$ be a random sample from $N(0, \sigma)$, and let $\bar{X}, S^2$ represent the sample mean and sample variance of this sample. Answer the following questions:

a. For what value of c the expression $c\frac{\bar{X}^2}{S^2}$ follows the F distribution? What are the degrees of freedom?

b. Using only your statistical tables (please no calculator!) find the 80th percentile of the distribution of $c\frac{\bar{X}^2}{S^2}$. Show all your work.