Exercise 1
Let $X_1, \ldots, X_n$ be a random sample from an exponential distribution with parameter $\theta$. Answer the following questions: Derive a likelihood ratio test for testing $H_0 : \theta = \theta_0$ against $H_a : \theta \neq \theta_0$, and show that the rejection region is of the form $\{\hat{X} \exp(-\theta_0 \hat{X}) \} < c$. Suppose $\alpha = 0.05$ and $\theta_0 = 1$. Explain why $c$ should be chosen so that $P(\hat{X} \exp(-\hat{X}) < c) = 0.05$.

Exercise 2
Suppose $X_1, \ldots, X_n$ are i.i.d. Poisson($\lambda_1$) and $Y_1, \ldots, Y_n$ are i.i.d. Poisson($\lambda_2$). The two samples are independent. Find the most powerful test for testing $H_0 : \lambda_1 = \lambda_2 = 2$ against $H_a : \lambda_1 = \frac{1}{2}, \lambda_2 = 3$.

Exercise 3
Suppose $X_1, \ldots, X_n$ are i.i.d. Poisson($\lambda$). Answer the following questions:

a. If $\lambda_a > \lambda_0$ find the form of the best critical region for testing $H_0 : \lambda = \lambda_0$ against $H_a : \lambda = \lambda_a$.

b. Explain how you would actually find the critical region of the test derived in (a).

c. Is the test found in (a) uniformly most powerful?

Exercise 4
Suppose $X_1, \ldots, X_n$ are i.i.d. $N(\mu, \sigma^2)$ and assume that the mean is unknown. Derive the likelihood ratio test for testing $H_0 : \sigma^2 = \sigma^2_0$ against $H_a : \sigma^2 > \sigma^2_0$ and show that the likelihood ratio test is equivalent of the test statistic $\frac{(n-1)s^2}{\sigma^2_0}$.