EXERCISE 1

Part A:
It is claimed that the histogram below shows the distribution of the sample mean $\bar{x}$, when repeated samples of size $n = 36$ are selected with replacement from the population $(2.6, 2.8, 3.0, 3.2, 3.4)$. Clearly explain if there is anything wrong with this histogram.

Part B:

a. What distribution does the sum (total) of 36 observations selected from the same population as above follow?

b. Sketch the histogram (roughly) of the total of repeated samples (with replacement) of size 36 selected from the above population. Make sure that you mark off some important values on the horizontal axis.

EXERCISE 2

A local bakery uses its stock of sugar according to demand for its products. Indeed, the weekly sugar use follows the normal distribution with mean 2700 lb. and standard deviation 400 lb. The starting supply of sugar is 4000 lb. and there is a regularly scheduled weekly delivery of 2500 lb. Find the probability that, after 12 weeks, the supply of sugar will be below 2000 lb.

EXERCISE 3

Let $X_1, X_2, \cdots, X_5$ be a random sample of size 5 from $N(0, 1)$. Let $X_6$ be another independent observation from the same population.

a. What is the distribution of $W = \sum_{i=1}^{5} X_i^2$?

b. What is the distribution of $U = \sum_{i=1}^{5} (X_i - \bar{X})^2$?

c. What is the distribution of $\sum_{i=1}^{5} (X_i - \bar{X})^2 + X_6^2$?
EXERCISE 4
Let $Z_1, Z_2, \cdots, Z_{16}$ be a random sample of size 16 from the standard normal distribution $N(0, 1)$. Let $X_1, X_2, \cdots, X_{64}$ be a random sample of size 64 from the normal distribution $N(\mu, 1)$. The two samples are independent.

a. Find $P(Z_1 > 2)$.
b. Find $P(\sum_{i=1}^{16} Z_i > 2)$.
c. Find $P(\sum_{i=1}^{16} Z_i^2 > 6.91)$.
d. Let $S^2$ be the sample variance of the first sample. Find $c$ such that $P(S^2 > c) = 0.05$.
e. What is the distribution of $Y$, where $Y = \sum_{i=1}^{16} Z_i^2 + \sum_{i=1}^{64} (X_i - \mu)^2$?
f. Find $EY$.
g. Find $Var(Y)$.
h. Approximate $P(Y > 105)$.

EXERCISE 5
Let $X_1, X_2, \cdots, X_{25}$ be a random sample of size 25 from $N(6, 2)$, and let $Y_1, Y_2, \cdots, Y_{35}$ be a random sample of size 35 from $\sim N(10, 5)$. The two samples are independent.

a. Find the distribution of $X_1 + Y_1$.
b. Find $P(\bar{X} > 6.8)$.
c. What is the distribution of $\sum_{i=1}^{25} (\frac{X_i - 6}{2})^2$?
d. What is the distribution of $W$, where $W = \sum_{i=1}^{25} (\frac{X_i - 6}{2})^2 + \sum_{i=1}^{35} (\frac{Y_i - 10}{5})^2$?
e. Use the exact distribution of $W$ to find $b$ such that $P(W > b) = 0.05$.
f. Use the limiting distribution of $W$ to approximate $b$ such that $P(W > b) = 0.05$.
g. Let $s_X^2$ and $s_Y^2$ be the sample variances from the two samples above. Find $c_1, c_2, c_3, c_4$ such that the following expression follows the $\chi^2$ distribution: $\frac{c_1 s_X^2}{c_3} + \frac{c_2 s_Y^2}{c_4}$.
h. Let $Q$ be a random variable with moment generating function $M_Q(t) = (1 - 2t)^{-20}$. If $Q$ and $W$ are independent, what is the distribution of $Q + W$?
i. Let $U$ be another random variable with moment-generating function $M_U(t) = e^{5000t+50000t^2}$. Find $P(27100 < (U - 500)^2 < 50200)$. 