University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Practice 1

EXERCISE 1
Find the distribution of the random variable X for each of the following moment-generating functions:

a. $M_X(t) = \left[\frac{1}{3}e^t + \frac{2}{3}\right]^5$.

b. $M_X(t) = \frac{e^t}{2-e^t}$.

c. $M_X(t) = e^{2(e^t-1)}$.

EXERCISE 2
Let $M_X(t) = \frac{1}{6}e^t + \frac{2}{6}e^{2t} + \frac{3}{6}e^{3t}$ be the moment-generating function of a random variable X.

a. Find $E(X)$.

b. Find $var(X)$.

c. Find the distribution of X.

EXERCISE 3
Let X follow the Poisson probability distribution with parameter λ. Its moment-generating function is $M_X(t) = e^{\lambda(e^t-1)}$.

a. Show that the moment-generating function of $Z = \frac{X-\lambda}{\sqrt{\lambda}}$ is given by:

$$M_Z(t) = e^{-\sqrt{\lambda}t}e^{\lambda(e^{\frac{t}{\sqrt{\lambda}}}-1)}.$$

b. Use the series expansion of

$$e^{\frac{t}{\sqrt{\lambda}}} = 1 + \frac{\frac{t}{\sqrt{\lambda}}}{1!} + \frac{(\frac{t}{\sqrt{\lambda}})^2}{2!} + \frac{(\frac{t}{\sqrt{\lambda}})^3}{3!} + \cdots$$

to show that

$$\lim_{\lambda\to\infty} M_Z(t) = e^{\frac{1}{2}t^2}.$$

In other words, as $\lambda \to \infty$, the ratio $Z = \frac{X-\lambda}{\sqrt{\lambda}}$ converges to the standard normal distribution.

EXERCISE 4
Use the result of part (b) of the previous exercise:
In the interest of pollution control an experimenter wants to count the number of bacteria per small volume of water. Let X denote the bacteria count per cubic centimeter of water, and assume that X follows the Poisson distribution with parameter $\lambda = 100$. If the allowable pollution in a water supply is a count of 110 bacteria per cubic centimeter, approximate the probability that X will be at most 110.

EXERCISE 5

Let $X_1, X_2, \cdots, X_n$ be i.i.d. random sample from $N(\mu, \sigma)$. Using moment genearating functions show that the sum of these n observations $T = \sum_{i=1}^{n} X_i$ also follows the normal distribution. What is the mean and standard deviation of T?

EXERCISE 6

Suppose that $X_1, \cdots, X_m$ and $Y_1, \cdots, Y_n$ are two samples, with $X \sim N(\mu_1, \sigma_1)$ and $Y \sim N(\mu_2, \sigma_2)$. The difference between the sample means, $\bar{X} - \bar{Y}$, is then a linear combination of $m + n$ normal random variables.

a. Find $E(\bar{X} - \bar{Y})$.

b. Find $Var(\bar{X} - \bar{Y})$.

c. Use moment generating functions to show that the distribution of $\bar{X} - \bar{Y}$ is normal with mean and variance equal to your answers in (a) and (b).

d. Suppose $\sigma_1^2 = 2, \sigma_2^2 = 2.5$, and $m = n$. Find the sample sizes so that $\bar{X} - \bar{Y}$ will be within one unit of $\mu_1 - \mu_2$ with probability 0.95.

EXERCISE 7

If the random variable X follows the normal distribution with $\mu = 0$, $\sigma^2 = 1$ and $Y = e^X$ find the probability density of Y. This is called the lognormal distribution.

EXERCISE 8

If the radius of a circle X is an exponential random variable with parameter λ, find the probability density function of its area Y.