University of California, Los Angeles
Department of Statistics

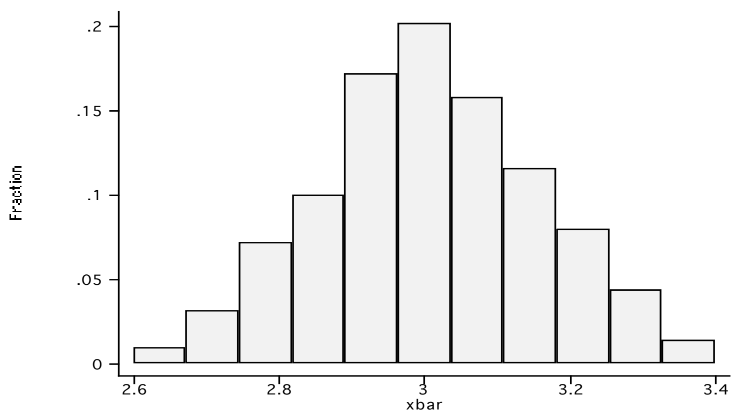
Statistics 100B **Instructor: Nicolas Christou**

Practice 2

EXERCISE 1
Part A:
It is claimed that the histogram below shows the distribution of the sample mean $\bar{x}$, when repeated samples of size $n = 36$ are selected with replacement from the population $(2.6, 2.8, 3.0, 3.2, 3.4)$. Clearly explain if there is anything wrong with this histogram.
Part B:

a. What distribution does the sum (total) of 36 observations selected from the same population as above follow?

b. Sketch the histogram (roughly) of the total of repeated samples (with repcalcement) of size 36 selected from the above population. Make sure that you mark off some important values on the horizontal axis.



EXERCISE 2
A local bakery uses its stock of sugar according to demand for its products. Indeed, the weekly sugar use follows the normal distribution with mean 2700 lb. and standard deviation 400 lb. The starting supply of sugar is 4000 lb. and there is a regularly scheduled weekly delivery of 2500 lb. Find the probability that, after 12 weeks, the supply of sugar will be below 2000 lb.

EXERCISE 3
Answer the following questions:

a. Let X and Y be independent normal random variables, each with mean μ and standard deviation σ. Consider the random quantities $X + Y$ and $X - Y$. Find the moment generating function of $X + Y$ and the moment generating function of $X - Y$.

b. Refer to part (b). Find now the joint moment generating function of $(X + Y, X - Y)$.

c. Are $X + Y$ and $X - Y$ independent? Explain your answer using moment generating functions.

EXERCISE 4

Let $\mathbf{X} = (X_1, X_2, X_3)'$ be a random vector with $var(\mathbf{X}) = \begin{pmatrix} 5 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 3 \end{pmatrix}$

a. Find te variance of $X_1 - 2X_2 + X_3$.

b. Let $Y_1 = X_1 + X_2$ and $Y_2 = X_1 + X_2 + X_3$. Find the variance covariance matrix of $\mathbf{Y} = (Y_1, Y_2)$.

EXERCISE 5

Consider the game of roulette at a casino. As a reminder a roulette has 38 numbers (1-36 plus 0 and 00). Let's say that you want to bet on number 13. If you win it pays $35:1$ which means that you get your \$1 back plus \$35.

a. Write the expression that computes the exact probability that In 10000 plays the casino will make more than \$400? (This involves the binomial distribution). Approximate this probability using normal approximation to binomial.

b. Compute the probability of question (a) using the central limit theorem. *Hint:* The number of plays can be viewed as a random sample from a population that has mean μ and standard deviation σ. First find this population and then compute μ and σ.

EXERCISE 6

Let $(X_i, Y_i), i = 1, 2, \ldots, n$, be a random sample from a bivariate normal distribution. Find the joint moment generating function of $(\bar{X}, \bar{Y})$. What is the distribution of $(\bar{X}, \bar{Y})$. Note: $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ are independent.

EXERCISE 7

Let X_1, X_2, X_3 be i.i.d. $N(0, 1)$ and let $Y_1 = \frac{X_1+X_2+X_3}{\sqrt{3}}$, $Y_2 = \frac{X_1-X_2}{\sqrt{2}}$, $Y_3 = \frac{X_1+X_2-2X_3}{\sqrt{6}}$. Show that Y_1, Y_2, Y_3 are i.i.d. $N(0, 1)$.

EXERCISE 8

Let $\mathbf{X} = (X_1, X_2, X_3)$ has joint moment generating function

$$M_{\mathbf{X}}(t_1, t_2, t_3) = (1 - t_1 + 2t_2)^{-4}(1 - t_1 + 3t_3)^{-3}(1 - t_1)^{-2}.$$

Answer the following questions:

a. Find the moment generating function of (X_1, X_3).

b. Find the moment generating function of X_1.

c. Find the moment generating function of X_3.

d. Are X_1, X_3 independent?

e. Find the moment generating function of (X_2, X_3).

f. Are X_2, X_3 independent?