EXERCISE 1
Suppose \((Y_1, Y_2, \ldots, Y_n)\)' is a random vector with mean \(\mu_1\) and variance covariance matrix \(\sigma^2 V\), where \(V\) is an \(n \times n\) symmetric matrix of known constants. Consider the expressions
\[(a) \ m = \frac{1'}{V^{-1}}Y \quad \text{and} \quad \frac{1'}{V^{-1}}Y_1 \quad \text{and} \quad (b) \ q = \frac{1'}{n}(Y - m_1)\.
Find the mean and variance of (a) and the mean of (b).

EXERCISE 2
Answer the following questions:
\[a. \ \text{Suppose } Y \sim \Gamma(\alpha, \beta) \text{ distribution. Find the mean and variance covariance matrix of the random vector } X = e^Y.\]
\[b. \ \text{Let } X_1, X_2, \ldots, X_n \text{ be i.i.d random variables with } X_i \sim \exp(\lambda). \text{ Find the expected value and variance of } \frac{1}{X}. \text{ where } \bar{X} \text{ is the sample mean of } X_1, X_2, \ldots, X_n.\]

EXERCISE 3
Suppose \(Y_1, \ldots, Y_n\) are i.i.d. random variables with \(Y_i \sim N(\mu, \sigma^2)\). Express the following vector in the form \(AY\) and find its mean and variance:
\[
\begin{pmatrix}
\bar{Y} \\
Y_1 - Y_2 \\
Y_2 - Y_3 \\
\vdots \\
Y_{n-1} - Y_n
\end{pmatrix}
\]

EXERCISE 4
Answer the following questions:
\[a. \ \text{Let } X \sim \Gamma(\frac{n}{2}, \beta). \text{ Find the distribution of } Y = \frac{2X}{\beta} \text{ using the method of cdf and the method of moment generating functions.}\]
\[b. \ \text{Suppose } X \text{ has the p.d.f. } f(x) = 4x^3, 0 < x < 1. \text{ Use the method of cdf to show that } Y = -2\ln X^4 \text{ follows a gamma distribution. What are the parameters of this gamma distribution?}\]

EXERCISE 5
Suppose that \(X_1, \ldots, X_m\) and \(Y_1, \ldots, Y_n\) are two samples, with \(X_i \sim N(\mu_1, \sigma_1)\) and \(Y_i \sim N(\mu_2, \sigma_2)\). The difference between the sample means, \(\bar{X} - \bar{Y}\), is then a linear combination of \(m + n\) normal random variables. All the random variables are independent. Answer the following questions:
\[a. \ \text{Use moment generating functions to show that } X - Y \text{ follows a normal distribution. Find the mean and variance of this distribution.}\]
\[b. \ \text{Suppose } \sigma_1^2 = 2, \sigma_2^2 = 2.5, \text{ and } m = n. \text{ Find the sample size } n \text{ so that } \bar{X} - \bar{Y} \text{ will be within one unit of } \mu_1 - \mu_2 \text{ with probability 0.95. You can use the standard normal table from the course website here: http://www.stat.ucla.edu/~nchristo/statistics100B/stat100b_z_table.pdf.}\]