University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Quiz 1

Answer the following questions:

a. Suppose $X \sim \Gamma(\frac{1}{2}, 2)$. Find the pdf of $Y = X^{\frac{1}{4}}$.

b. Suppose $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with $X_i \sim exp(\lambda)$. Find the distribution of $\sum_{i=1}^n X_i$ and then find $E\left[\frac{1}{\sum_{i=1}^n X_i}\right]$.

c. Suppose $X \sim \Gamma(\alpha, \beta)$. Use the theorem for exponential families to find the mean and variance of X.

d. Let X and Y be independent normal random variables, each with mean μ and standard deviation σ.

1. Consider the random quantities $X+Y$ and $X-Y$. Find the moment generating function of $X+Y$ and the moment generating function of $X-Y$.
2. Find the joint moment generating function of $(X+Y, X-Y)$.
3. Are $X+Y$ and $X-Y$ independent? Explain your answer using moment generating functions.

e. Let X_1, X_2, X_3 be i.i.d. random variables $N(0,1)$. Show that $Y_1 = X_1 + \delta X_3$ and $Y_2 = X_2 + \delta X_3$ have bivariate normal distribution. Find the value of δ so that the correlation coefficient between Y_1 and Y_2 is $\rho = \frac{1}{2}$.

f. Expectation of a quadratic form using properties of the trace of a square matrix: Give a numerical example of your choice to explain the expectation of a quadratic form $\mathbf{X}'\mathbf{A}\mathbf{X}$, where $\mathbf{X}$ is a 3×1 random vector and $\mathbf{A}$ is a 3×3 symmetric matrix.

g. Suppose 3 random variables follow jointly a multivariate normal distribution as follows:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} \sim N_3\left(\begin{bmatrix} 170 \\ 68 \\ 40 \end{bmatrix}, \begin{bmatrix} 400 & 64 & 128 \\ 64 & 16 & 0 \\ 128 & 0 & 256 \end{bmatrix} \right)$$

Given that $Y_2 = 72$ and $Y_3 = 24$ find the probability that Y_1 exceeds 200.

h. Week 1 lectures notes: Please summarize in less than 1 page the topic of moment generating functions for a single random variable. Use your class notes to provide the definition, examples, properties, how we use them in distribution theory for sum of independent random variables, and how to find moments using moment generating functions.

i. Week 2 lecture notes: Please summarize in less than 1 page the topic of joint moment generating functions. Give the definition, properties, independence, moments, and as an example use the multivariate normal distribution.

j. Consider the ratio of two independent random variables $\frac{Q_1}{Q_2}$. Suppose $Q_! \sim \Gamma(\alpha_1, \beta_1)$ and $Q_2 \sim \Gamma(\alpha_2, \beta_2)$. Find the mean and variance of $\frac{Q_1}{Q_2}$.