**University of California, Los Angeles**
**Department of Statistics**

**Statistics 100B** **Instructor: Nicolas Christou**

## Quiz 2

Answer the following questions:

a. We discussed in class today the multinomial probability distribution and its joint moment generating function. Here is a note on the multinomial distribution: A sequence of $n$ independent experiments is performed and each experiment can result in one of $r$ possible outcomes with probabilities $p_1, p_2, \ldots, p_r$ with $\sum_{i=1}^r p_i = 1$. Let $X_i$ be the number of the $n$ experiments that result in outcome $i$, $i = 1, 2, \ldots, r$. Then, $P(X_1 = x_1, X_2 = x_2, \ldots, X_r = x_r) = \frac{n!}{n_1! n_2! \cdots n_r!} p_1^{x_1} p_2^{x_2} \cdots p_r^{x_r}$. The joint moment generating function of the multinomial distribution is given by $M_{\mathbf{X}}(\mathbf{t}) = (p_1 e^{t_1} + p_2 e^{t_2} + \ldots + p_r e^{t_r})^n$. Use properties of joint moment generating functions to find the probability distribution of $X_1$.

b. Refer to question (a). Use the joint moment generating function of the multinomial distribution and the theorem and corollary on handout #10, page 1 to find the mean and variance of $X_1$.

c. Refer to question (a). Show that $\text{cov}(X_i, X_j) = -np_i p_j$. Give an intuitive explanation of the negative sign.

d. Let $X$ and $Y$ be independent normal random variables, each with mean $\mu$ and standard deviation $\sigma$.

1. Consider the random quantities $X + Y$ and $X - Y$. Find the moment generating function of $X + Y$ and the moment generating function of $X - Y$.
2. Find the joint moment generating function of $(X + Y, X - Y)$.
3. Are $X + Y$ and $X - Y$ independent? Explain your answer using moment generating functions.