

University of California, Los Angeles
Department of Statistics

Statistics 100B

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Quiz 3

Answer the following questions:

- a. If Y_1 and Y_2 are random variables such that $X_1 = Y_1 + Y_2$ and $X_2 = Y_1 - Y_2$ are independent $N(0, 1)$ random variables, show that Y_1 and Y_2 have a bivariate normal distribution. Find the mean and variance covariance matrix of $\mathbf{Y} = (Y_1, Y_2)'$.
- b. Let X_1, X_2, X_3 be i.i.d. $N(0, 1)$ random variables. Suppose $Y_1 = X_1 + X_2 + X_3, Y_2 = X_1 - X_2, Y_3 = X_1 - X_3$. Find the joint pdf of $\mathbf{Y} = (Y_1, Y_2, Y_3)'$ using:
 1. The method of variable transformations (Jacobian).
 2. Multivariate normal distribution properties.
- c. Let X_1, X_2, X_3 be i.i.d. random variables $N(0, 1)$. Show that $Y_1 = X_1 + \delta X_3$ and $Y_2 = X_2 + \delta X_3$ have bivariate normal distribution. Find the value of δ so that the correlation coefficient between Y_1 and Y_2 is $\rho = \frac{1}{2}$.