University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Quiz 6

EXERCISE 1
A probability problem.
Let's Make a Deal! A player is asked to choose one of three doors. Behind one of the doors there is a prize. Suppose the player chose door 1. The host of the game who knows where the prize is opens door 3 and the player sees that there is no prize behind this door. Then the host asks the player: Would you like to switch to door 2 or stay with door 1? Define the following events: H_i : Host opens door i and D_i : Prize is behind door i. Find $P(D_2|H_3)$ and $P(D_1|H_3)$ to show that switching has higher probability.

EXERCISE 2
Let $X_1, X_2, \ldots, X_n$ be independent exponential random variables with mean $i\theta$. For example, $E(X_1) = \theta, E(X_2) = 2\theta$, etc. Suppose an estimate of of θ is $\hat{\theta} = \sum_{i=1}^{n}(\frac{X_i}{ni})$.

a. Find the distribution of $\hat{\theta}$.
b. Find $E[\hat{\theta}^{-1}]$.
c. Find the MSE of $c\hat{\theta}^{-1}$ as an estimator of θ^{-1}, and find c that minimizes that MSE.

EXERCISE 3
Answer the following questions:

a. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with $X_i \sim Poisson(\lambda)$. Show that $\hat{\theta}_1 = \frac{1}{n}\sum_{i=1}^{n} X_i(X_i - 1)$ is unbiased estimate of λ^2.
b. Let $X_1, X_2, \ldots, X_n$ i.i.d. normal random variables with $X_i \sim N(0, \sigma)$. Consider $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2$. Is $\hat{\sigma}^2$ an efficient estimator of σ^2? First find the information using two different methods, for example, the variance of the score function and $-E\left[\frac{\partial^2 lnf(x;\theta)}{\partial\theta^2}\right]$.
c. Let $X_1, \ldots, X_n$ i.i.d. random variables with $X_i \sim N(\mu, \sigma)$. Is $\bar{X}$ a consistent estimator of μ? What if $var(X_i) = \sigma^2$ and $cov(X_i, X_j) = \rho\sigma^2$? Is $\bar{X}$ a consistent estimator of μ?