University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Quiz 7

EXERCISE 1
Let $Y_1, \ldots, Y_n$ be random variables with $E(Y_i) = \mu$, $\mathrm{var}(\mathrm{Y}_i) = \sigma^2$, and $cov(Y_i, Y_j) = \rho\sigma^2$. The variance covariance matrix is of the form $(a-b)\mathbf{I} + b\mathbf{J}$, where $a = 1, b = \rho, \mathbf{J} = \mathbf{1}\mathbf{1}'$. In our model $\mathbf{\Sigma} = \sigma^2\left[(1-\rho)\mathbf{I} + \rho\mathbf{J}\right]$. The inverse of this special matrix can be obtained as follows: $\mathbf{\Sigma^{-1}} = \frac{1}{\sigma^2(1-\rho)}\left[\mathbf{I} - \frac{\rho}{1+(n-1)\rho}\mathbf{J}\right]$. Suppose the estimator of μ is given by $\hat{\mu} = \frac{\mathbf{1'\Sigma^{-1}Y}}{\mathbf{1'\Sigma^{-1}1}}$. Is $\hat{\mu}$ unbiased estimator of μ? Show that $var(\hat{\mu}) = \frac{1}{\mathbf{1'\Sigma^{-1}1}}$ and simplify it using the inverse of the variance covariance matrix given above. Explain why $\rho > -\frac{1}{n-1}$.

EXERCISE 2
Let $Y_1, Y_2, \ldots, Y_n$ independent random variables, and let $Y_i \sim N(i\theta, i\sigma)$, i.e. $E(Y_i) = i\theta$ and $var(Y_i) = i^2\sigma^2$, for $i = 1, 2, \ldots, n$. Find the maximum likelihood estimator of θ. Is this estimator efficient estimator of θ?

EXERCISE 3
Consider the two samples $X_1, \ldots, X_n$ i.i.d. $N(\mu_1, \sigma)$ and $Y_1, \ldots, Y_m$ i.i.d. $N(\mu_2, \sigma)$. If $\mu_1 = \mu_2 = \mu$ find the MLEs for μ and σ^2.

EXERCISE 4
Suppose $Y_i = \beta_1 x_i + \epsilon_i$. The x_i's are not random and $\epsilon_1, \ldots, \epsilon_n$ are independent with $E(\epsilon_i) = 0, \mathrm{var}(\epsilon_i) = \sigma^2$. Assume also that $\epsilon_i \sim N(0, \sigma)$. Find $\hat{\beta}_1$ and $\hat{\sigma}^2$, the maximum likelihood estimates of β_1 and σ^2. Find the expected value and variance of $\hat{\beta}_1$ and its distribution.