Confidence intervals for the population mean $\mu$:

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>Sample size $n$</th>
<th>Assumption on population</th>
<th>Confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Known</td>
<td>$n \geq 30$</td>
<td>Any</td>
<td>$\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$</td>
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Sample size determination for $\mu$ estimation:

$$n = \left( \frac{z_{\alpha/2} \sigma}{E} \right)^2,$$
where $E$ is the error of estimation.

Confidence interval for the population proportion $p$:

$$\frac{x}{n} - z_{\alpha/2} \sqrt{\frac{\frac{x}{n}(1 - \frac{x}{n})}{n}} \leq p \leq \frac{x}{n} - z_{\alpha/2} \sqrt{\frac{\frac{x}{n}(1 - \frac{x}{n})}{n}}$$

Where

- $\frac{x}{n}$ Sample proportion ($x$ successes in $n$ trials)
- $n$ Sample size

Sample size determination for $p$ estimation:

$$n = \frac{z_{\alpha/2}^2 p(1 - p)}{E^2},$$
where $E$ is the error of estimation.

If no information on $p$, use $p = 0.5$.
If there is information on $p$ use it.
Confidence interval for the population variance $\sigma^2$:

$$\frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2};n-1}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2};n-1}}$$

Where $s^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$ is the sample variance of $n$ observations. You can easier compute the sample variance by expanding the previous formula to get $s^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n}x_i^2 - \frac{(\sum_{i=1}^{n}x_i)^2}{n}\right]$. Note that $\chi^2_{1-\frac{\alpha}{2};n-1}$ is the value of a $\chi^2$ random variable with $n-1$ degrees of freedom such that the area to its left is $1-\frac{\alpha}{2}$ and $\chi^2_{\frac{\alpha}{2};n-1}$ is the value of a $\chi^2$ random variable with $n-1$ degrees of freedom such that the area to its left is $\frac{\alpha}{2}$.

Confidence interval for the difference between two population means $\mu_1 - \mu_2$ when $\sigma_1^2, \sigma_2^2$ are known:

$$\bar{x}_1 - \bar{x}_2 - z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq \bar{x}_1 - \bar{x}_2 + z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Where $\bar{x}_1, \bar{x}_2$ are the sample means of two samples independently selected from two populations with means $\mu_1, \mu_2$ and variances $\sigma_1^2, \sigma_2^2$ respectively.

Confidence interval for the difference between two normal population means $\mu_1 - \mu_2$ when $\sigma_1^2 = \sigma_2^2$ but unknown:

$$\bar{x}_1 - \bar{x}_2 - t_{\frac{\alpha}{2};n_1+n_2-2}\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \leq \mu_1 - \mu_2 \leq \bar{x}_1 - \bar{x}_2 + t_{\frac{\alpha}{2};n_1+n_2-2}\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Where $s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$ is the pooled sample variance (the estimate of the true but unknown common population variance $\sigma^2$). This confidence interval is based on the fact that $\frac{(n_1+n_2-2)s^2}{\sigma^2} \sim \chi^2_{n_1+n_2-2}$.

Confidence interval for the difference between two population proportions $p_1 - p_2$:

$$\frac{x_1}{n_1} - \frac{x_2}{n_2} - z_{\frac{\alpha}{2}}\sqrt{\frac{\frac{x_1(1-x_1)}{n_1} + \frac{x_2(1-x_2)}{n_2}}{n_1} + \frac{z_{\frac{\alpha}{2}}(1-x_1)}{n_1}} \leq p_1 - p_2 \leq \frac{x_1}{n_1} - \frac{x_2}{n_2} + z_{\frac{\alpha}{2}}\sqrt{\frac{\frac{x_1(1-x_1)}{n_1} + \frac{x_2(1-x_2)}{n_2}}{n_1} + \frac{z_{\frac{\alpha}{2}}(1-x_1)}{n_1}}$$

Where $x_1$ is the number of successes among $n_1$ trials with probability of success $p_1$, and $x_2$ is the number of successes among $n_2$ trials with probability of success $p_2$.

Confidence interval for the ratio of two normal population variances $\frac{\sigma_1^2}{\sigma_2^2}$:

$$\frac{s_1^2}{s_2^2} \frac{1}{F_{1-\frac{\alpha}{2};n_1-1,n_2-1}} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{s_1^2}{s_2^2} F_{1-\frac{\alpha}{2};n_2-1,n_1-1}$$

Where $s_1^2, s_2^2$ are the sample variances based on two independent samples of size $n_1, n_2$ selected from two normal populations $N(\mu_1, \sigma_1)$ and $N(\mu_2, \sigma_2)$. 