University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Homework 10

Exercise 1

Suppose X has the possible values 0,1,2,3,4. Suppose that the null hypothesis says that X is uniform on these integers, while the alternative hypothesis says that $X \sim b(4, \frac{1}{2})$. Let's see what happens if we let k of the Neyman-Pearson lemma be equal to 0.6. Complete the next table and find the best critical region when $k = 0.6$ and compute the power of the test.

x	0	1	2	3	4
$P(X = x \vert H_0)$					
$P(X = x \vert H_a)$					
$\frac{P(X=x \vert H_0)}{P(X=x \vert H_a)}$					

Exercise 2

Let X be a random variable whose probability mass function under H_0 and H_a is given by

x	1	2	3	4	5	6	7
$f(x \vert H_0)$	0.01	0.01	0.01	0.01	0.01	0.01	0.94
$f(x \vert H_a)$	0.06	0.05	0.04	0.03	0.02	0.01	0.79

Use the Neyman-Pearson lemma to find the most powerful test for testing H_0 against H_a with $\alpha = 0.04$. Compute the probability of Type II error for this test.

Exercise 3

It is known that the random variable X has a probability density function of the form $f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$. We want to test the hypothesis $H_0 : \theta = 2$ against $H_a : \theta = 4$. A random sample X_1, X_2 of size $n = 2$ will be selected. Suppose the rejection region is given by $X_1 + X_2 > 9.49$. Calculate the Type I and Type II error probabilities. For the Type I error probability you should use your χ^2 table. For the Type II error probability you may find the following table useful. Please show all your work.

χ^2	$P(\chi^2_{df} < 9.49)$	$P(\chi^2_{df} < \frac{9.49}{2})$	$P(\chi^2_{df} < 2 \times 9.49)$
χ^2_2	0.9913	0.9068	0.9999
χ^2_4	0.9500	0.6855	0.9992
χ^2_8	0.6973	0.2155	0.9850

Exercise 4

A coin is thrown independently 10 times to test that the probability of heads is $\frac{1}{2}$ against the alternative that the probability is not $\frac{1}{2}$. The test rejects H_0 if either 0 or 10 heads are observed.

a. What is the significance level α of the test?

b. If in fact the probability of heads is 0.1, what is the power of the test?

Exercise 5
Let X be a uniform random variable on $(0, \theta)$. You have exactly one observation from this distribution and you want to test the null hypothesis $H_0 : \theta = 10$ against the alternative $H_a : \theta > 10$, and you want to use significance level $\alpha = 0.10$. Two testing procedures are being considered:
Procedure G rejects H_0 if and only if $X \geq 9$.
Procedure K rejects H_0 if either $X \geq 9.5$ or if $X \leq 0.5$.

a. Confirm that Procedure G has a Type I error probability of 0.10.

b. Confirm that Procedure K has a Type I error probability of 0.10.

c. Find the power of Procedure G when $\theta = 12$.

d. Find the power of Procedure K when $\theta = 12$.

Exercise 6
Answer the following questions:

a. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a Poisson distribution with parameter λ. Find the best critical region for testing
$H_0 : \lambda = 2$
$H_a : \lambda = 5$
using the Neyman-Pearson lemma.

b. Let $Y_1, Y_2, \ldots, Y_n$ be the outcomes of n independent Bernoulli trials. Find the best critical region for testing
$H_0 : p = p_0$
$H_a : p > p_0$
using the Neyman-Pearson lemma.

Exercise 7
Let $X_1, \ldots, X_n$ be a random sample from an exponential distribution with parameter θ. Answer the following questions: Derive a likelihood ratio test for testing $H_0 : \theta = \theta_0$ against $H_a : \theta \neq \theta_0$, and show that the rejection region is of the form $\{\bar{X} exp(-\theta_0 \bar{X})\} < c$. Suppose $\alpha = 0.05$ and $\theta_0 = 1$. Explain why c should be chosen so that $P(\bar{X} exp(-\bar{X}) < c) = 0.05$.

Exercise 8
Suppose $X_1, \ldots, X_n$ are i.i.d. Poisson(λ_1) and $Y_1, \ldots, Y_n$ are i.i.d. Poisson(λ_2). The two samples are independent. Find the most powerful test for testing $H_0 : \lambda_1 = \lambda_2 = 2$ against $H_a : \lambda_1 = \frac{1}{2}, \lambda_2 = 3$.