EXERCISE
Let $Y_1, Y_2, \ldots, Y_n$ independent random variables, and let $Y_i \sim N(i\theta, i\sigma^2)$, i.e. $E(Y_i) = i\theta$ and $\text{var}(Y_i) = i^2\sigma^2$, for $i = 1, 2, \ldots, n$. Find the maximum likelihood estimator of $\theta$. Is this estimator efficient estimator of $\theta$?

EXERCISE 2
If $X$ is binomial $(n, p)$, then the variance of $\hat{p} = \frac{X}{n}$ (which is the maximum likelihood estimate of $p$) is $\sigma^2_{\hat{p}} = \frac{p(1-p)}{n}$. This variance is often estimated by $\hat{\sigma}^2 = \frac{\hat{p}(1-\hat{p})}{n} = \frac{X(1-X)}{n}$. Is this an unbiased estimator of $\sigma^2_{\hat{p}}$? If not find a constant $c$ so that $c\hat{\sigma}^2$ is unbiased.

EXERCISE 3
The numbers $w_1, w_2, \ldots, w_n$ are known positive values. The random variables $X_1, X_2, \ldots, X_n$ are independent, and the distribution of $X_i$ is $N(\mu, \sigma^2\sqrt{w_i})$. Both parameters $\mu$ and $\sigma$ are unknown. Find the maximum likelihood estimates of $\mu$ and $\sigma$.

EXERCISE 4
Suppose $Y_1, Y_2, \ldots, Y_n$ follow multivariate normal with mean $\mu\mathbf{1}$ and variance covariance matrix $\sigma^2\mathbf{V}$, where $\mathbf{V}$ is an $n \times n$ symmetric matrix of known constants. Show that the maximum likelihood estimates of $\mu$ and $\sigma^2$ are $\hat{\mu} = \frac{\mathbf{1}'\mathbf{V}^{-1}\mathbf{Y}}{\mathbf{1}'\mathbf{V}^{-1}\mathbf{1}}$ and $\hat{\sigma}^2 = \frac{(\mathbf{Y} - \hat{\mu}\mathbf{1})'\mathbf{V}^{-1}(\mathbf{Y} - \hat{\mu}\mathbf{1})}{n}$. Find $E(\hat{\mu})$ and $E(\hat{\sigma}^2)$. 