EXERCISE 1
Let $X_1, \ldots, X_n$ be i.i.d. $U(0, \alpha)$ and let $X_{(1)}, \ldots, X_{(n)}$ be the order statistics. Define the range as $R = X_{(n)} - X_{(1)}$ and the midrange as $Q = \frac{X_{(1)} + X_{(n)}}{2}$. Find the joint pdf of $R, Q$.

EXERCISE 2
Let $X_1, \ldots, X_n$ be i.i.d. random variables $N(\mu, \sigma)$. Let $S^2$ be the usual unbiased estimator of $\sigma^2$. Find a function of $S^2$, say $g(S^2)$ such that $Eg(S^2) = \sigma$.

EXERCISE 3
Let $X_1, \ldots, X_n$ be i.i.d. random variables with $X_i \sim N(\mu, \sigma)$. Consider the signal-to-noise ratio $\frac{\mu}{\sigma}$. A natural estimator of this ratio is given by $\frac{\bar{X}}{S}$. Is this an unbiased estimator of the signal-to-noise ratio? If not adjust it to be unbiased.

EXERCISE 4
Let $X_1, X_2, \ldots, X_n$ denote an i.i.d. random sample from the following distribution ($\alpha > 0$).

$$f(x) = \begin{cases} \frac{\alpha x^{\alpha-1}}{3^\alpha}, & 0 \leq x \leq 3 \\ 0, & \text{elsewhere} \end{cases}$$

Find the expected value of $X$.
Derive the method of moments estimator of $\alpha$.
Derive the method of maximum likelihood estimate of $\alpha$.

EXERCISE 5
Let $X_1, \ldots, X_n$ be i.i.d. random variables with $X_i \sim U(0, 1)$. Show that the $j$th order statistic follows the beta distribution, $\text{Beta}(j, n - j + 1)$.

EXERCISE 6
Let $Y(1) < Y(2) < Y(3) < Y(4)$ denote the order statistics of a random sample of size $n = 4$ from a distribution having p.d.f. $f(x) = 2x, 0 < x < 1$. Compute $P(Y(3) > \frac{1}{2})$.

EXERCISE 7
Let $Y(1) < Y(2) < Y(3)$ denote the order statistics of a random sample of size $n = 3$ from a distribution with p.d.f. $f(x) = 1, 0 < x < 1$. Find the p.d.f. of the range $Q = Y(3) - Y(1)$. 