University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Homework 2

EXERCISE 1

The two theorems we discussed in class are:

$$E\left(\sum_{i=1}^{k} \frac{\partial w_i(\boldsymbol{\theta})}{\partial \theta_j} t_i(x)\right) = -\frac{\partial}{\partial \theta_j} logc(\boldsymbol{\theta}).$$

and

$$var\left(\sum_{i=1}^{k} \frac{\partial w_i(\boldsymbol{\theta})}{\partial \theta_j} t_i(x)\right) = -\frac{\partial^2}{\partial \theta_j^2} logc(\boldsymbol{\theta}) - E\left(\sum_{i=1}^{k} \frac{\partial^2 w_i(\boldsymbol{\theta})}{\partial \theta_j^2} t_i(x)\right).$$

Note: Here log is the natural logarithm.

In class we showed that the binomial pmf can be expressed in the exponential family form and then we found $E(X) = np$ using the first theorem. Use the second theorem to show that $var(X) = np(1-p)$.

EXERCISE 2

Prove theorem 1 using the following:

$$\begin{aligned} \int_x f(x|\boldsymbol{\theta}) dx &= 1 \\ \int_x h(x) c(\boldsymbol{\theta}) exp\left(\sum_{i=1}^{k} w_i(\boldsymbol{\theta}) t_i(x)\right) &= 1. \end{aligned}$$

Hint: Differentiate both sides w.r.t. θ_j and rearrange to prove theorem 1.