EXERCISE 1
The two theorems we discussed in class are:

\[
E \left( \sum_{i=1}^{k} \frac{\partial w_i(\theta)}{\partial \theta_j} t_i(x) \right) = -\frac{\partial}{\partial \theta_j} \log c(\theta) .
\]

and

\[
\text{var} \left( \sum_{i=1}^{k} \frac{\partial w_i(\theta)}{\partial \theta_j} t_i(x) \right) = -\frac{\partial^2}{\partial \theta_j^2} \log c(\theta) - E \left( \sum_{i=1}^{k} \frac{\partial^2 w_i(\theta)}{\partial \theta_j^2} t_i(x) \right).
\]

Note: Here log is the natural logarithm.

In class we showed that the binomial pmf can be expressed in the exponential family form and then we found \( E(X) = np \) using the first theorem. Use the second theorem to show that \( \text{var}(X) = np(1-p) \).

EXERCISE 2
Prove theorem 1 using the following:

\[
\int_x f(x|\theta) dx = 1
\]

\[
\int_x h(x)c(\theta)\exp \left( \sum_{i=1}^{k} w_i(\theta) t_i(x) \right) = 1.
\]

Hint: Differentiate both sides w.r.t. \( \theta_j \) and rearrange to prove theorem 1.