University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Homework 3

EXERCISE 1

Let $(X_1, Y_1), \ldots, (X_n, Y_n)$, be a random sample from a bivariate normal distribution with parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho$. (Note: $(X_1, Y_1), \ldots, (X_n, Y_n)$ are independent). What is the joint distribution of $(\bar{X}, \bar{Y})$? Hint: Find the joint moment generating function of $(\bar{X}, \bar{Y})$ and compare it to the joint moment generating function of multivariate normal distribution.

EXERCISE 2

Find the moment generating function of the Bernoulli random variable. Then use the fact that the sum of independent Bernoulli is binomial to find the moment generating function of the binomial distribution.

EXERCISE 3

Answer the following questions:

a. Let X_1, X_2, X_3 be i.i.d. random variables $N(0,1)$. Show that $Y_1 = X_1 + \delta X_3$ and $Y_2 = X_2 + \delta X_3$ have bivariate normal distribution. Find the value of δ so that the correlation coefficient between Y_1 and Y_2 is $\rho = \frac{1}{2}$.

b. Let X and Y follow the bivariate normal distribution with parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho$. Show that $W = X - \mu_1$ and $Q = (Y - \mu_2) - \rho\frac{\sigma_2}{\sigma_1}(X - \mu_1)$ are independent normal random variables.

EXERCISE 4

Answer the following questions:

a. Let X_1 and X_2 be two independent normal random variables with mean zero and variance 1. Show that the vector $\mathbf{Z} = (Z_1, Z_2)'$, where

$$\begin{aligned} Z_1 &= \mu_1 + \sigma_1 X_1 \\ Z_2 &= \mu_2 + \rho\sigma_2 X_1 + \sigma_2\sqrt{1-\rho^2}X_2 \end{aligned}$$

follows the bivariate normal distribution with parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho$.

b. Suppose $\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N_2(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Consider the vector $\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$, where $Y_i = e^{X_i}, i = 1, 2$. Find EY_1^3 and covariance between Y_1^3, Y_2^3.