University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Homework 3

EXERCISE 1
Let $X \sim N(\mu, \sigma)$.

a. Use the properties of moment generating functions to show that $aX+b \sim N(a\mu+b, a\sigma)$.

b. Use the cdf method to show that $aX + b \sim N(a\mu + b, a\sigma)$.

EXERCISE 2
Let $\ln(X) \sim N(\mu, \sigma)$. Find EX and $var(X)$.

EXERCISE 3
Let $X_1, X_2, , X_n$ be i.i.d. random variables with $X_i \sim \Gamma(\alpha, \beta)$. Use the properties of moment generating functions to find the distribution of $T = X_1 + X_2 + \ldots X_n$ and $\bar{X} = \frac{X_1+X_2+\ldots X_n}{n}$.

EXERCISE 4
Let $X \sim N(\mu, \sigma)$. Stein's lemma states that if g is a differentiable function satisfying $Eg'(X) < \infty$ then $E\left[g(X)(X-\mu)\right] = \sigma^2 Eg'(X)$. Use Stein's lemma to show that $EX^4 = \mu^4 + 6\mu^2\sigma^2 + 3\sigma^4$. Hint: Write EX^4 as $EX^3(X - \mu + \mu)$.

EXERCISE 5
Let $M_{X1,X_2}(t_1, t_2)$ be the moment generating function of the bivariate normal distribution. Show that if we let $\Psi(t_1, t_2) = lnM_{X1,X_2}(t_1, t_2)$ then $\frac{\partial^2 \Psi(0,0)}{\partial t_1 \partial t_2}$ gives the covariance directly. Can we also obtain the covariance using $\frac{\partial^2 M_{X_1,X_2}(0,0)}{\partial t_1 \partial t_2} - \frac{\partial M_{X_1,X_2}(0,0)}{\partial t_1} \times \frac{\partial M_{X_1,X_2}(0,0)}{\partial t_2}$?