University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Homework 3

EXERCISE 1
Let X and Y be independent normal random variables, each with mean μ and standard deviation σ.

a. Consider the random quantities $X+Y$ and $X-Y$. Find the moment generating function of $X+Y$ and the moment generating function of $X-Y$.

b. Find the joint moment generating function of $(X+Y, X-Y)$.

c. Are $X+Y$ and $X-Y$ independent? Explain your answer using moment generating functions.

EXERCISE 2
Let X and Y be independent standard normal random variables. Consider the transformation $U=X+Y$ and $V=X-Y$.

a. Use the theory of distributions of functions of random variables (Jacobian) to find the joint pdf of U and V. Are U and V independent? Why? What is the distribution of U and the distribution of V?

b. Use moment generating functions to answer question (a).

EXERCISE 3
Let $X \sim \text{beta}(\alpha, \beta)$ and Let $Y \sim \text{beta}(\alpha+\beta, \gamma)$. If X and Y are independent random variables find the joint pdf of U and V where $U=XY$ and $V=X$.

EXERCISE 4
Suppose Y is the number of pollution particles in volume v and assume that Y follows the Poisson distribution with mean λv.

a. If a point is randomly selected within the volume v find the pdf of its distance R from the nearest pollution particle. Note the volume of a sphere with radius r is $\frac{4}{3}\pi r^3$.

b. Refer to question (a). Let $U=R^3$. Show that U follows exponential distribution. What is the mean and variance of U?

EXERCISE 5
Let X_1, X_2, and X_3 be three independent gamma random variables with parameters $(\frac{r_1}{2}, 2), (\frac{r_2}{2}, 2), (\frac{r_3}{2}, 2)$ respectively. Use the joint distribution of functions of random variables and the Jacobian to show that $Y_1 = \frac{X_1}{X_2}$ and $Y_2 = X_1 + X_2$ are independent and that $Y_2 \sim \Gamma(\frac{r_1+r_2}{2}, 2)$. Conclude by reasoning that $\frac{\frac{X_1}{r_1}}{\frac{X_2}{r_2}}$ and $\frac{\frac{X_3}{r_3}}{\frac{X_1+X_2}{r_1+r_2}}$ are independent.