University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Homework 4

Answer the following questions:

a. Derive the distribution of the sample mean $\bar{X}$ of independent $X_1, \ldots, X_n$ where, $X_i \sim \Gamma(\alpha, \beta)$. Find a transformation of $\bar{X}$ that follows a χ^2 distribution. What are the degrees of freedom of this transformation?

b. Suppose X has a uniform distribution on $(0, 1)$. Find the mean and variance covariance matrix of the random vector $\begin{pmatrix} X \\ X^2 \end{pmatrix}$.

c. Suppose X_1 and X_2 are independent with $\Gamma(\alpha, 1)$ and $\Gamma(\alpha + \frac{1}{2}, 1)$ distributions. Let $Y = 2\sqrt{X_1 X_2}$. Find EY and $var(Y)$.

d. Let $(X_1, Y_1), \ldots, (X_n, Y_n)$, be a random sample from a bivariate normal distribution, $N(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. (Note: $(X_1, Y_1), \ldots, (X_n, Y_n)$ are independent). What is the distribution of $n \left(\bar{X} - \mu_1, \bar{Y} - \mu_2\right) \mathbf{\Sigma^{-1}} \begin{pmatrix} \bar{X} - \mu_1 \\ \bar{Y} - \mu_2 \end{pmatrix}$. Hint: First find the joint distribution of $(\bar{X}, \bar{Y})$.