University of California, Los Angeles Department of Statistics

Statistics 100B

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Homework 6

Answer the following questions:

- a. Let X_1, X_2, \ldots, X_n be i.i.d.Poisson (λ) and let \overline{X} and S^2 be the sample mean and sample variance respectively. Each one of these two estimators has expected value equal to λ (why?). Which estimator is better?
- b. Let X_1, \ldots, X_n be i.i.d. $N(\theta, \theta), \theta > 0$. For this model both \bar{X} and cS are unbiased estimators of θ , where $c = \frac{\sqrt{n-1}\Gamma(\frac{n-1}{2})}{\sqrt{2}\Gamma(\frac{n}{2})}$. Show that for any α the estimator $\alpha \bar{X} + (1-\alpha)cS$ is also unbiased estimator of θ . For what value of α this estimator has the minimum variance?
- c. Let X_1, \ldots, X_n be i.i.d. random variables with $X_i \sim \Gamma(\alpha, \beta)$ with α known. Find an unbiased estimator of $\frac{1}{\beta}$. (Find $E\frac{1}{X}$ and then adjust it to be unbiased of $\frac{1}{\beta}$.)
- d. Let X_1, X_2, X_3, X_4 be i.i.d. random variables with $X_i \sim N(0, 1)$. Show that

$$\sum_{i=1}^{4} (X_i - \bar{X})^2 = \frac{(X_1 - X_2)^2}{2} + \frac{[X_3 - \frac{(X_1 + X_2)}{2}]^2}{\frac{3}{2}} + \frac{[X_4 - \frac{(X_1 + X_2 + X_3)}{3}]^2}{\frac{4}{3}}.$$

Show that the three terms in the RHS are independent each with a χ_1^2 distribution.