EXERCISE 1
Let $Y_1, \ldots, Y_n$ be i.i.d. random variables with pdf $f(y|\theta) = \left(\frac{2y}{\theta}\right) exp\left(-\frac{y^2}{\theta}\right)$, $y > 0$ (Weibull distribution). Show that $\hat{\theta} = \frac{\sum_{i=1}^{n} Y_i^2}{n}$ is an unbiased estimator of $\theta$. Is $\hat{\theta}$ an efficient estimator of $\theta$?

EXERCISE 2
Let $X_1, \ldots, X_n$ be i.i.d. $N(\theta, \theta)$, $\theta > 0$. For this model both $\bar{X}$ and $cS$ are unbiased estimators of $\theta$, where $c = \sqrt{\frac{n-1}{\Gamma\left(\frac{n-1}{2}\right)\sqrt{2\Gamma\left(\frac{n}{2}\right)}}}$. Define the estimator $T = \alpha_1 \bar{X} + \alpha_2 (cS)$, where we do not assume that $\alpha_1 + \alpha_2 = 1$. Find the estimator that minimizes $E(T - \theta)^2$.

EXERCISE 3
Answer the following questions:

a. If $X_1, \ldots, X_n$ are independent random variables with $X_i \sim N(\mu, \sigma^2)$ and if $r = [(X_1 - \mu_1)^2 + \ldots + (X_n - \mu_n)^2]^{\frac{1}{2}}$, show that $E(r) = \sqrt{2\sigma^2 \Gamma\left(\frac{n+\frac{1}{2}}{2}\right)}$.

b. Consider the following two independent sets of random variables: Let $X_1, \ldots, X_n$ i.i.d. random variables with $X_i \sim N(\mu_1, \sigma)$ and let $Y_1, \ldots, Y_m$ i.i.d. random variables with $Y_i \sim N(\mu_2, \sigma)$. Consider the random variable $S^2_p = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2 + \sum_{i=1}^{m}(Y_i - \bar{Y})^2}{n+m-2}$. Find $E S^2_p$.

EXERCISE 4
Answer the following questions:

a. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a normal population with mean zero and unknown variance $\sigma^2$. Consider the estimator $\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} X_i^2$. Is it efficient?

b. Consider the following two independent samples $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ selected from two exponential distributions with parameters $\lambda_1$ and $\lambda_2$. Find $c$ such that the distribution of $c \bar{X}$ is $F$. What are the degrees of freedom?

EXERCISE 5
Suppose $Y_1, Y_2, \ldots, Y_n$ follow multivariate normal with mean $\mu 1$ and variance covariance matrix $\sigma^2 V$, where $V$ is an $n \times n$ symmetric matrix of known constants. Show that the maximum likelihood estimates of $\mu$ and $\sigma^2$ are $\hat{\mu} = \frac{1}{n} V^{-1} Y$ and $\hat{\sigma}^2 = \frac{1}{n} (Y - \hat{\mu} 1)' V^{-1} (Y - \hat{\mu} 1)$. Find the information matrix $I(\theta)$, where $\theta = (\mu, \sigma^2)'$. Is $\hat{\mu}$ an efficient estimator of $\mu$?
EXERCISE 6
Let $X_1, X_2, \cdots, X_n$ denote an i.i.d. random sample from the following distribution ($\alpha > 0$).

$$f(x) = \begin{cases} \frac{\alpha x^{\alpha-1}}{3^\alpha}, & 0 \leq x \leq 3 \\ 0, & \text{elsewhere} \end{cases}$$

Find the expected value of $X$.
Derive the method of moments estimator of $\alpha$.
Derive the method of maximum likelihood estimate of $\alpha$.

EXERCISE 7
Let $X_1, \ldots, X_n$ be i.i.d. $U(0, \alpha)$ and let $X_{(1)}, \ldots, X_{(n)}$ be the order statistics. Define the range as $R = X_{(n)} - X_{(1)}$ and the midrange as $Q = \frac{X_{(1)} + X_{(n)}}{2}$. Find the joint pdf of $R, Q$.

EXERCISE 8
Let $X_1, \ldots, X_n$ be i.i.d. random variables with $X_i \sim N(\mu, \sigma)$. Find $c$ so that $E[cS^2 - \sigma^2]^2$ is minimized. How is this new estimator $cS^2$ compared to $S^2$?