University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Homework 7

Answer the following questions:

a. Suppose $Y_1, Y_2, \ldots, Y_n$ follow multivariate normal with mean $\mu\mathbf{1}$ and variance covariance matrix $\sigma^2\mathbf{V}$, where $\mathbf{V}$ is an $n \times n$ symmetric matrix of known constants. Show that the maximum likelihood estimates of μ and σ^2 are $\hat{\mu} = \frac{\mathbf{1'V^{-1}Y}}{\mathbf{1'V^{-1}1}}$ and $\hat{\sigma}^2 = \frac{(\mathbf{Y}-\hat{\mu}\mathbf{1})'\mathbf{V^{-1}}(\mathbf{Y}-\hat{\mu}\mathbf{1})}{n}$.

b. Refer to question (a). Find $E(\hat{\mu})$ and $E(\hat{\sigma}^2)$.

c. Refer to question (a). Find the Fisher information matrix $\mathbf{I}(\boldsymbol{\theta})$, where $\boldsymbol{\theta} = (\mu, \sigma^2)'$. Is $\hat{\mu}$ an efficient estimator of μ?

d. Let $Y_1, Y_2, \ldots, Y_n$ independent random variables, and let $Y_i \sim N(i\theta, i\sigma)$, i.e. $E(Y_i) = i\theta$ and $var(Y_i) = i^2\sigma^2$, for $i = 1, 2, \ldots, n$. Find the maximum likelihood estimator of θ. Is this estimator efficient estimator of θ?

e. Suppose that the radius of a circle is measured with an error $\epsilon \sim N(0, \sigma)$. If n independent measurements are made find an unbiased estimator of the area of the circle.

f. Let $Y_1, \ldots, Y_n$ be i.i.d. random variables from the Weibull distribution $f(y|\theta) = (\frac{2y}{\theta})exp(-\frac{y^2}{\theta}), y > 0$. Show that $\hat{\theta} = \frac{\sum_{i=1}^n Y_i^2}{n}$ is unbiased estimator of θ. Is $\hat{\theta}$ an efficient estimator of θ?

g. Let $X_1, \ldots, X_n$ be i.i.d. $N(\theta, \theta), \theta > 0$. For this model both $\bar{X}$ and cS are unbiased estimators of θ, where $c = \frac{\sqrt{n-1}\Gamma(\frac{n-1}{2})}{\sqrt{2}\Gamma(\frac{n}{2})}$. Define the estimator $T = \alpha_1\bar{X} + \alpha_2(cS)$, where we do not assume that $\alpha_1 + \alpha_2 = 1$. Find the estimator that minimizes $E(T-\theta)^2$.