University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Homework 8

EXERCISE 1
Let $X_1, \ldots, X_n$ be i.i.d. random variables with $X_i \sim N(\mu, \sigma)$. Answer the following questions:

a. Verify that $\hat{\theta} = \bar{X}S^2$ is an unbiased estimator of $\theta = \mu\sigma^2$. Find the variance of $\hat{\theta}$.

b. Find an unbiased estimator of $\theta = \frac{\mu}{\sigma^2}$. Find the variance of this estimator.

EXERCISE 2
Let $X_1, \ldots, X_n$ be i.i.d. from $N(\theta, 1)$ and let $U_1, \ldots, U_n$ be i.i.d. from $U(0, 1)$. All the $2n$ random variables are independent. Let $Y_i = X_i U_i, i = 1, \ldots, n$. If the X_i and U_i are both observed, then $\bar{X}$ would be a natural estimator for θ. If only the products $Y_1, \ldots, Y_n$ are observed, then $2\bar{Y}$ may be a reasonable estimator for θ. Are the two estimators unbiased? Determine the relative efficiency of $2\bar{Y}$ with respect to $\bar{X}$. Which estimator is more efficient?

EXERCISE 3
Let $X_1, \ldots, X_n$ be i.i.d. $N(0, \sigma)$. Show that $\hat{\theta} = \frac{C}{n}\sum_{i=1}^n \sqrt{X_i^2}$ is a consistent estimator of σ if and only if $C = \sqrt{\frac{\pi}{2}}$. Show that the MLE of σ is given by $\hat{\sigma} = \sqrt{\frac{1}{n}\sum_{i=1}^n X_i^2}$ and determine the asymptotic relative efficiency of $\hat{\theta}$ with $C = \sqrt{\frac{\pi}{2}}$ compared to $\hat{\sigma}$ the MLE of σ. Hint: Use the asymptotic efficiency of maximum likelihood estimates.

EXERCISE 4
Let $X_1, \ldots, X_m$ be i.i.d. random variables from an exponential distribution with parameter λ_x and $Y_1, \ldots, Y_n$ be i.i.d. random variables from an exponential distribution with parameter λ_y. The two samples are independent. Answer the following questions.

a. Find an unbiased estimator of the ratio $\frac{\lambda_x}{\lambda_y}$. Your answer should be a function of $\sum_{i=1}^m X_i$ and $\sum_{i=1}^n Y_i$.

b. Use MSE to find the best estimator of $\frac{\lambda_x}{\lambda_y}$ of the form $\hat{\delta} = c\frac{\bar{Y}}{\bar{X}}$.

EXERCISE 5
Let $X_1, \ldots, X_n$ be i.i.d. Poisson random variables with parameter λ. Use the definition of sufficiency to show that $\sum_{i=1}^n X_i$ is a sufficient statistic for λ.

EXERCISE 6
Let $X_1, \ldots, X_n$ be i.i.d. random variables with $f(x) = \frac{2x}{\theta}e^{-\frac{x^2}{\theta}}, x > 0$. Use the factorization theorem to show that $\sum_{i=1}^n X_i^2$ is a sufficient statistic for θ.