

University of California, Los Angeles  
Department of Statistics

Statistics 100B

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Homework 8

Answer the following questions:

- a. Consider the regression model through the origin  $y_i = \beta_1 x_i + \epsilon_i$ , where  $\epsilon_i \sim N(0, \sigma)$ . It is assumed that the regression line passes through the origin  $(0, 0)$ . Find the MLE of  $\sigma^2$ , its expectation, and finally adjust it to be unbiased.
- b. Refer to question (a). Show that  $\frac{(n-1)s_e^2}{\sigma^2} \sim \chi_{n-1}^2$ , where  $s_e^2$  is the unbiased estimator of  $\sigma^2$  from question (a).
- c. Refer to question (a). Find the distribution of  $s_e^2$ .
- d. Let  $X_1, X_2, \dots, X_n$  denote an i.i.d. random sample from the following distribution ( $\alpha > 0$ ).

$$f(x) = \begin{cases} \frac{\alpha x^{\alpha-1}}{3^\alpha}, & 0 \leq x \leq 3 \\ 0, & \text{elsewhere} \end{cases}$$

Find the expected value of  $X$ .

Derive the method of moments estimator of  $\alpha$ .

Derive the method of maximum likelihood estimate of  $\alpha$ .

- e. Consider the regression model  $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ , where  $\epsilon_i \sim N(0, \sigma)$ . Find the Fisher information matrix  $I(\beta_0, \beta_1, \sigma^2)$ . Are  $\hat{\beta}_0$  and  $\hat{\beta}_1$  efficient estimators of  $\beta_0$  and  $\beta_1$ ?