EXERCISE 1
Let $X_1, \ldots, X_n$ be i.i.d. random variables with $X_i \sim N(\mu, \sigma)$. Answer the following questions:

a. Verify that $\hat{\theta} = \bar{X}S^2$ is an unbiased estimator of $\theta = \mu \sigma^2$. Find the variance of $\hat{\theta}$.

b. Find an unbiased estimator of $\theta = \mu \sigma^2$. Find the variance of this estimator.

EXERCISE 2
Let $X_1, \ldots, X_n$ be i.i.d. from $N(\theta, 1)$ and let $U_1, \ldots, U_n$ be i.i.d. from $U(0, 1)$. All the $2n$ random variables are independent. Let $Y_i = X_i U_i, i = 1, \ldots, n$. If the $X_i$ and $U_i$ are both observed, then $\bar{X}$ would be a natural estimator for $\theta$. If only the products $Y_1, \ldots, Y_n$ are observed, then $2\bar{Y}$ may be a reasonable estimator for $\theta$. Are the two estimators unbiased? Determine the relative efficiency of $2\bar{Y}$ with respect to $\bar{X}$. Which estimator is more efficient?

EXERCISE 3
Let $X_1, \ldots, X_n$ be i.i.d. $N(0, \sigma)$. Show that $\hat{\theta} = C n \sum_{i=1}^{n} \sqrt{X_i^2}$ is a consistent estimator of $\sigma$ if and only if $C = \sqrt{\frac{\pi}{2}}$. Show that the MLE of $\sigma$ is given by $\hat{\sigma} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} X_i^2}$ and determine the asymptotic relative efficiency of $\hat{\theta}$ with $C = \sqrt{\frac{\pi}{2}}$ compared to $\hat{\sigma}$ the MLE of $\sigma$. Hint: Use the asymptotic efficiency of maximum likelihood estimates.

EXERCISE 4
Let $X_1, \ldots, X_m$ be i.i.d. random variables from an exponential distribution with parameter $\lambda_x$ and $Y_1, \ldots, Y_n$ be i.i.d. random variables from an exponential distribution with parameter $\lambda_y$. The two samples are independent. Answer the following questions.

a. Find an unbiased estimator of the ratio $\frac{\lambda_x}{\lambda_y}$. Your answer should be a function of $\sum_{i=1}^{m} X_i$ and $\sum_{i=1}^{n} Y_i$.

b. Use MSE to find the best estimator of $\frac{\lambda_x}{\lambda_y}$ of the form $\hat{\delta} = c \bar{X}$.

EXERCISE 5
Let $X_1, \ldots, X_n$ be i.i.d. Poisson random variables with parameter $\lambda$. Use the definition of sufficiency to show that $\sum_{i=1}^{n} X_i$ is a sufficient statistic for $\lambda$.

EXERCISE 6
Let $X_1, \ldots, X_n$ be i.i.d. random variables with $f(x) = \frac{2x}{\theta} e^{-x^2/\theta}, x > 0$. Use the factorization theorem to show that $\sum_{i=1}^{n} X_i^2$ is a sufficient statistic for $\theta$. 