

Homework 9

EXERCISE 1

Let X_1, \dots, X_n be i.i.d. random variables with $X_i \sim N(\mu, \sigma)$. Use the Lehmann-Scheffé theorem to show that $\sum_{i=1}^n X_i$ and $\sum_{i=1}^n X_i^2$ are minimal sufficient statistic for μ and σ^2

EXERCISE 2

Suppose that two independent random samples of n_1 and n_2 observations are selected from normal populations with means μ_1, μ_2 and variances σ_1^2, σ_2^2 respectively. Find a confidence interval for the variance ratio $\frac{\sigma_1^2}{\sigma_2^2}$ with confidence level $1 - \alpha$.

EXERCISE 3

The sample mean \bar{X} is a good estimator of the population mean μ . It can also be used to predict a future value of X independently selected from the population. Assume that you have a sample mean \bar{x} and a sample variance s^2 , based on a random sample of n measurements from a normal population. Construct a prediction interval for a new observation X , say X_0 . Use $1 - \alpha$ confidence level. Hint: Start with $X_0 - \bar{X}$ and then use t distribution.

EXERCISE 4

(From *Mathematical Statistics and Data Analysis*, by J. Rice, 2nd Edition.)

In a study done at the National Institute of Science and Technology (Steel et al. 1980), asbestos fibers on filters were counted as part of a project to develop measurement standards for asbestos concentration. Asbestos dissolved in water was spread on a filter, and punches of 3-mm diameter were taken from the filter and mounted on a transmission electron microscope. An operator counted the number of fibers in each of 23 grid squares, yielding the following counts:

31	29	19	18	31	28
34	27	34	30	16	18
26	27	27	18	24	22
28	24	21	17	24	

Assume that the Poisson distribution with unknown parameter λ would be a plausible model for describing the variability from grid square to grid square in this situation.

1. Use the method of maximum likelihood to estimate the parameter λ .
2. Use the asymptotic properties of the maximum likelihood estimates to construct a 95% confidence interval for λ . As a reminder, for large samples the distribution of $\frac{\hat{\theta} - \theta}{\sqrt{\frac{1}{nI(\theta)}}}$ is approximately standard normal, where $I(\theta)$ is the Fisher information.

EXERCISE 5

Let X_1, \dots, X_n be i.i.d. random variables with $f(x) = \frac{2x}{\theta} e^{-\frac{x^2}{\theta}}$, $x > 0$. Use the Lehmann-Scheffé theorem to show that $\sum_{i=1}^n X_i^2$ is a minimal sufficient statistic for θ . Is the sufficient statistic obtained using the factorization theorem the same?

EXERCISE 6

Let X_1, X_2, \dots, X_9 and Y_1, Y_2, \dots, Y_{12} represent two independent random samples from the respective normal distributions $N(\mu_1, \sigma_1)$ and $N(\mu_2, \sigma_2)$. It is given that $\sigma_1^2 = 3\sigma_2^2$, but σ_2^2 is unknown. Define a random variable which has a t distribution and use it to find a 95% confidence interval for $\mu_1 - \mu_2$.