University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Homework 9

EXERCISE 1
Let $X_1, \ldots, X_n$ be i.i.d. random variables with $X_i \sim N(\mu, \sigma)$. Use the Lehmann-Scheffé theorem to show that $\sum_{i=1}^n X_i$ and $\sum_{i=1}^n X_i^2$ are minimal sufficient statistic for μ and σ^2

EXERCISE 2
Suppose that two independent random samples of n_1 and n_2 observations are selected from normal populations with means μ_1, μ_2 and variances σ_1^2, σ_2^2 respectively. Find a confidence interval for the variance ratio $\frac{\sigma_1^2}{\sigma_2^2}$ with confidence level $1 - \alpha$.

EXERCISE 3
The sample mean $\bar{X}$ is a good estimator of the population mean μ. It can also be used to predict a future value of X independently selected from the population. Assume that you have a sample mean $\bar{x}$ and a sample variance s^2, based on a random sample of n measurements from a normal population. Construct a prediction interval for a new observation X, say X_0. Use $1 - \alpha$ confidence level. Hint: Start with $X_0 - \bar{X}$ and then use t distribution.

EXERCISE 4
(From *Mathematical Statistics and Data Analysis*, by J. Rice, 2nd Edition.)
In a study done at the National Institute of Science and Technology (Steel et al. 1980), asbestos fibers on filters were counted as part of a project to develop measurement standards for asbestos concentration. Asbestos dissolved in water was spread on a filter, and punches of 3-mm diameter were taken from the filter and mounted on a transmission electron microscope. An operator counted the number of fibers in each of 23 grid squares, yielding the following counts:

31	29	19	18	31	28
34	27	34	30	16	18
26	27	27	18	24	22
28	24	21	17	24	

Assume that the Poisson distribution with unknown parameter λ would be a plausible model for describing the variability from grid square to grid square in this situation.

1. Use the method of maximum likelihood to estimate the parameter λ.
2. Use the asymptotic properties of the maximum likelihood estimates to construct a 95% confidence interval for λ. As a reminder, for large samples the distribution of $\frac{\hat{\theta}-\theta}{\sqrt{\frac{1}{nI(\theta)}}}$ is approximately standard normal, where $I(\theta)$ is the Fisher information.

EXERCISE 5
Let $X_1, \ldots, X_n$ be i.i.d. random variables with $f(x) = \frac{2x}{\theta} e^{-\frac{x^2}{\theta}}, x > 0$. Use the Lehmann-Scheffé theorem to show that $\sum_{i=1}^n X_i^2$ is a minimal sufficient statistic for θ. Is the sufficient statistic obtained using the factorization theorem the same?

EXERCISE 6
Let $X_1, X_2, \cdots, X_9$ and $Y_1, Y_2, \cdots, Y_{12}$ represent two independent random samples from the respective normal distributions $N(\mu_1, \sigma_1)$ and $N(\mu_2, \sigma_2)$. It is given that $\sigma_1^2 = 3\sigma_2^2$, but σ_2^2 is unknown. Define a random variable which has a t distribution and use it to find a 95% confidence interval for $\mu_1 - \mu_2$.