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Statistics 100B **Instructor: Nicolas Christou**

Method of moments - Examples

Very simple!

The method of moments is based on the assumption that the sample moments are good estimates of the corresponding population moments.

Definition:

Population moments	Sample moments
$EX = \mu$ is the first population moment	$\bar{X} = \frac{1}{n}\sum_{i=1}^n X_i$ is the first sample moment.
EX^2 is the second population moment	$\frac{1}{n}\sum_{i=1}^n X_i^2$ is the second sample moment.
$\vdots$	$\vdots$
$\vdots$	$\vdots$
EX^k is the kth population moment	$\frac{1}{n}\sum_{i=1}^n X_i^k$ is the kth sample moment.

Therefore, $\bar{X} = \frac{1}{n}\sum_{i=1}^n X_i$ is a good estimator of $EX = \mu$. Similarly, $\frac{1}{n}\sum_{i=1}^n X_i^2$ is a good estimator of EX^2, etc.

Example 1:
Suppose $X_1, X_2, \cdots, X_n$ is a random sample from a Poisson distribution with mean λ. Find the moment estimator of λ.

Example 2:
Let X follow the uniform distribution on the interval $(0, \theta)$, and $X_1, X_2, \cdots, X_n$ denote i.i.d. random variables from this distribution. Find the method of moments estimator of θ.

Example 3:
If $X_1, X_2, \cdots, X_n$ denotes a random sample from $N(\mu, \sigma)$, find the method of moments estimators of μ and σ^2.

Example 4:
If $X_1, X_2, \cdots, X_n$ denotes a random sample from $N(0, \sigma)$, find the method of moments estimators of σ^2.

Example 5:
Let $X_1, \cdots, X_n$ denote a random sample from the probability density function $f(x;\theta) = (\theta+1)x^\theta, \quad 0 < x < 1, \quad \theta > -1$. Find the method of moments estimator of θ.