University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Multivariate normal - practice problems

EXERCISE 1

Let $(X_1, Y_1), \ldots, (X_n, Y_n)$, be a random sample from a bivariate normal distribution with parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho$. (Note: $(X_1, Y_1), \ldots, (X_n, Y_n)$ are independent). What is the joint distribution of $(\bar{X}, \bar{Y})$? Hint: Find the joint moment generating function of $(\bar{X}, \bar{Y})$ and compare it to the joint moment generating function of multivariate normal distribution.

EXERCISE 2

Answer the following questions:

a. Let X_1, X_2, X_3 be i.i.d. random variables $N(0, 1)$. Show that $Y_1 = X_1 + \delta X_3$ and $Y_2 = X_2 + \delta X_3$ have bivariate normal distribution. Find the value of δ so that the correlation coefficient between Y_1 and Y_2 is $\rho = \frac{1}{2}$.

b. Let X and Y follow the bivariate normal distribution with parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho$. Show that $W = X - \mu_1$ and $Q = (Y - \mu_2) - \rho\frac{\sigma_2}{\sigma_1}(X - \mu_1)$ are independent normal random variables.

EXERCISE 3

Answer the following questions:

a. Let X_1 and X_2 be two independent normal random variables with mean zero and variance 1. Show that the vector $\mathbf{Z} = (Z_1, Z_2)'$, where

$$\begin{aligned} Z_1 &= \mu_1 + \sigma_1 X_1 \\ Z_2 &= \mu_2 + \rho\sigma_2 X_1 + \sigma_2\sqrt{1-\rho^2}X_2 \end{aligned}$$

follows the bivariate normal distribution with parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho$.

b. Suppose $\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N_2(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Consider the vector $\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$, where $Y_i = e^{X_i}, i = 1, 2$. Find EY_1^3 and covariance between Y_1^3, Y_2^3.