Normal approximation to binomial

Suppose that $X$ follows the binomial distribution with parameters $n$ and $p$. We can approximate binomial probabilities as follows:

- Calculate $np$ and $n(1-p)$. If both are $\geq 5$ continue.
- Compute $\mu = np$ and $\sigma = \sqrt{np(1-p)}$.
- Now, let $k$ be one of the possible values of $X$ (remember, $X = 0, 1, 2, \cdots, n$). Here is how you can approximate binomial probabilities:

  - At least $k$ successes $P(X \geq k) = P(Z > \frac{k-0.5-\mu}{\sigma}) = \cdots$
  - More than $k$ successes $P(X > k) = P(Z > \frac{k+0.5-\mu}{\sigma}) = \cdots$
  - At most $k$ successes $P(X \leq k) = P(Z < \frac{k+0.5-\mu}{\sigma}) = \cdots$
  - Less than $k$ successes $P(X < k) = P(Z < \frac{k-0.5-\mu}{\sigma}) = \cdots$
  - Exactly $k$ successes $P(X = k) = P(\frac{k-0.5-\mu}{\sigma} < Z < \frac{k+0.5-\mu}{\sigma}) = \cdots$

- Some comments: The approximation is good if both $np$ and $n(1-p)$ are $\geq 5$. These 2 requirements hold if $n$ is large, or if $n$ is not very large but $p \approx 0.5$. The $\pm 0.5$ is the so called continuity correction and you should use it.
Some examples:

Example 1:
To avoid accusations of sexism in a college class equally populated by male and female students, the professor flips a fair coin to decide whether to call upon a male or female student to answer a question directed to the class. The professor will call upon a female student if a tails occurs. Suppose the professor does this 1000 times during the semester.

   a. What is the probability that he calls upon a female student at least 530 times?

   b. What is the probability that he calls upon a female student at most 480 times?

   c. What is the probability that he calls upon a female student exactly 510 times?

Example 2:
A manufacturing process produces semiconductor chips with a known failure rate 6.3%. Assume that chip failures are independent of one another. You will be producing 2000 chips tomorrow.

   a. Find the expected number of defective chips produced.

   b. Find the standard deviation of the number of defective chips.

   c. Find the probability (approximate) that you will produce less than 135 defects.