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Statistics 100B **Instructor: Nicolas Christou**

The paired t test

In many experiments the same variable is measured under two different conditions. For example, in clinical trials the participants may be evaluated at baseline and then evaluated again at the end of the treatment. For example, the blood pressure is measured for several patients before and after administration of certain drug. The difference between the value at baseline and the value at the end is computed for each participant as follows:

	Value at baseline	Value at the end	Difference
Subject 1	x_{1b}	x_{1a}	$d_1 = x_{1a} - x_{1b}$
Subject 2	x_{2b}	x_{2a}	$d_2 = x_{2a} - x_{2b}$
Subject 3	x_{3b}	x_{3a}	$d_3 = x_{3a} - x_{3b}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
Subject n	x_{nb}	x_{na}	$d_n = x_{na} - x_{nb}$

We then compute the sample mean and sample standard deviation of the differences.

$$\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n}, \qquad s_d^2 = \frac{\sum_{i=n}^{n}(d_i - \bar{d})^2}{n-1}$$

The hypothesis we want to test is:
$H_0 : \mu_d = d_0$
$H_a : \mu_d < d_0$ or $\mu_d > d_0$ or $\mu_d \neq d_0$

If we choose $d_0 = 0$ we are testing whether the before and after treatment are the same.

Test statistic:

$$t = \frac{\bar{d} - d_0}{\frac{s_d}{\sqrt{n}}}$$

Assumption: The differences are treated as a random sample from a normal distribution.

We reject H_0 if the t value falls in the rejection region which is based on the significance level α and $n-1$ degrees of freedom.

Example:
From *Mathematical Statistics and Data Analysis*, John Rice, Third Edition, Duxbury (2007).

To study the effect of cigarette smoking on platelet aggregation researchers drew blood samples from 11 individuals before and after they smoked a cigarette and measured the percentage of blood platelet aggregation. Platelets are involved in the formation of blood clots, and it is known that smokers suffer more from disorders involving blood clots than do nonsmokers. This study can be found in Levine, P. H. (1973). An acute effect of cigarette smoking on platelet function, *Circulation, 48*, 619-623 (see attached article).

Before	After	Difference
25	27	2
25	29	4
27	37	10
44	56	12
30	46	16
67	82	15
53	57	4
53	80	27
52	61	9
60	59	-1
28	43	15

Test the null hypothesis that the means before and after are the same. Use $\alpha = 0.05$.

From the study of the effect of cigarette smoking on platelet aggregation. Please see the article by Levine, P. H. (1973), "An acute effect of cigarette smoking on platelet function", *Circulation, 48*, 619-623. We want to test the null hypothesis that the means of blood platelet aggregation before and after smoking a cigarette are the same.

```
> a <- read.table("platelet_aggregation.txt", header=TRUE)
> a
    x  y  d
1  25 27  2
2  25 29  4
3  27 37 10
4  44 56 12
5  30 46 16
6  67 82 15
7  53 57  4
8  53 80 27
9  52 61  9
10 60 59 -1
11 28 43 15
```

Here are the variances for each column:

```
> var(a$d)
[1] 63.61818
> var(a$x)
[1] 243.7636
> var(a$y)
[1] 334.8727
```

Possible choices for test statistics:

$$Z = \frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}.$$

$$t \approx \frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}, \quad \text{d.f.} = \frac{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right]^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}, \quad \text{rounded to the nearest integer.}$$

$$t = \frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{s_{\text{pooled}}^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \quad \text{d.f.} = n_1 + n_2 - 2.$$

$$t = \frac{\bar{d} - d_0}{\frac{s_d}{\sqrt{n}}}, \quad \text{d.f.} = n - 1.$$

Why is the paired t test more appropriate, and what do we gain by using it instead of the other test statistics above?